

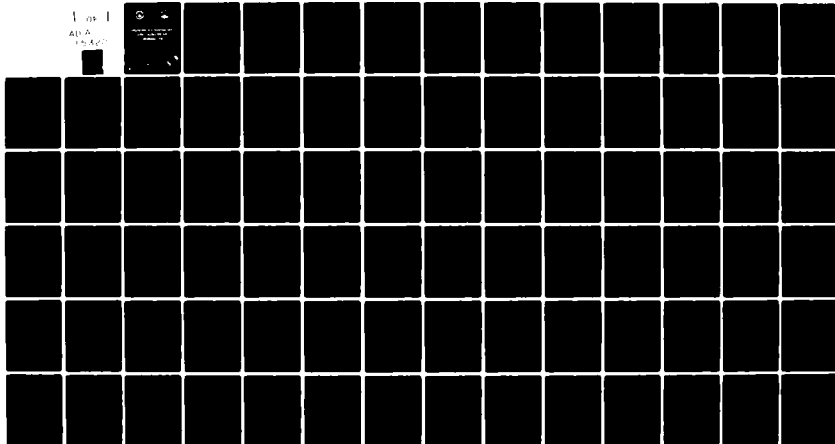
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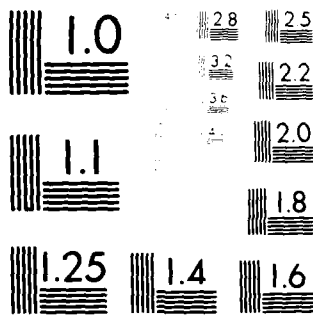
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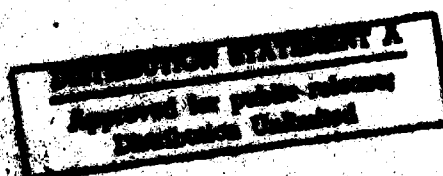
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EVALUATION OF A PROPOSED UICP LEVELS CALCULATION FOR CONSUMABLE ITEMS

OPERATIONS ANALYSIS DEPARTMENT

NAVY FLEET MATERIAL SUPPORT OFFICE
Mechanicsburg, Pennsylvania 17055

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EVALUATION OF A PROPOSED UICP LEVELS CALCULATION FOR CONSUMABLE ITEMS

PROJECT NO.
9322-D73-0092

REPORT 148

SUBMITTED:

Charles J. Bondi
C. J. BONDI
Operations Research Analyst

APPROVED:

R. A. Lippert
for R. A. LIPPERT, CDR, SC, USN
Director, Operations Analysis Department

J. B. Whittaker
J. B. WHITTAKER, CAPT, SC, USN
Commanding Officer, Navy Fleet
Material Support Office

DATE MAY 8 1982

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ABSTRACT

This study is a cost-benefit analysis of a proposed levels (order quantity and reorder level) calculation for consumable items. The Computation and Research Evaluation System (CARES) III Analyzer was used to evaluate the proposal. The input data consisted of samples of 1H and 1R Cognizance (COG) items. The criteria for evaluation were Total Variable Cost and Average Days Delay. Since the Navy imputes shortage cost from desired performance and budget levels, the relevant Total Variable Cost is the sum of procurement order cost and holding cost. Supply Material Availability was held constant at 85%. Additional cost projections were obtained concerning the implementation of a variable shortage cost.

This study makes the following recommendations regarding non-MARK O 1H and 1R cog items:

- Implement the proposed consumable levels formulas which include shortage costs in the order quantity calculation and make direct use of the probability of being out of stock at a random point in time.
- Use the Negative Binomial (vice Normal) distribution when computing the reorder level for 1R cog items whose leadtime demand is less than 20.
- Remove or relax unnecessary constraints on the order quantity and reorder level calculation.



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EXECUTIVE SUMMARY

1. Background. The current consumable Uniform Inventory Control Program (UICP) order quantity (Q) and reorder level (R) calculation employs approximations to avoid the iterative computations required by the mathematically exact formulas. In computing Q, the current formula omits the backorder terms. In computing R, the current formula approximates the optimal value of the probability of being out of stock during an order cycle (RISK) by the optimal probability of being out of stock at a random point in time (P_{out}). Naval Supply Systems Command (COMNAVSUPSYSCOM) proposed new levels formulas, which avoid the approximations and are, therefore, more accurate. This study evaluates the proposed formulas. The levels formulas were compared under several combinations of constraint settings and probability distribution policies. Hence, the findings include the impact of varying constraint usage and choice of probability distribution. Additional results include estimates of cost increases when shortage cost is considered to vary proportionally with replacement price (vice constant across a large group of items).

2. Objective. To evaluate the costs and benefits of replacing the current order quantity and reorder level formulas for consumable items by the proposed formulas.

3. Approach. The Computation and Research Evaluation System (CARES) III Analyzer was modified to permit comparisons between the current and proposed calculations and among the constraint and probability distribution policy alternatives. The input data consisted of samples of 1H cog and 1R cog items. MARK 0 items were excluded from the study. Results of the computer

model used in evaluating the calculations are Total Variable Cost (\$TVC), Average Inventory Investment $\left\{ \$ \left(\frac{Q}{2} + R \right) \right\}$, Average Days Delay (ADD), and Steady State Buys (SSBUYS). Since the Navy imputes shortage cost from desired performance and budget levels, the relevant Total Variable Cost is the sum of procurement order cost and holding cost. Supply Material Availability (SMA) was held constant at 85% to enable a comparison of the relevant costs.

An additional user-determined parameter (QBREAKPOINT) was established in order to deal with computational difficulties which occur when the proposed formulas are used with the Negative Binomial distribution. This parameter, when equalled or exceeded by Q, forces the use of the Normal distribution.

4. Findings. The preliminary findings established a method for avoiding computational difficulties when the proposed calculation was used. When Q is large and the probability distribution is Negative Binomial, the solution for R is infeasible. Hence the Negative Binomial distribution is used only when Q is less than a user-determined upper bound (QBREAKPOINT). The preliminary findings indicated that cost and benefit results were virtually insensitive as QBREAKPOINT varies between 15 and 40. For the remainder of the study, QBREAKPOINT was established as 30.

Regarding the levels formulas, the proposed levels calculation is more cost-effective than the current calculation under the current UICP and loosened constraints. In particular, TABLE I shows the results for the constrained case when both the Negative Binomial distribution (low demand items) and the Normal distribution (high demand items) are used.

TABLE I
IMPACT OF PROPOSED CALCULATIONS
(DOLLAR VALUES ARE IN \$MILLIONS)

	1H COG - 10,906 ITEMS				1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	$\% \Delta$	CURRENT	PROPOSED	Δ	$\% \Delta$
\$TVC	16.805	16.474	- .331	-2.0	38.902	38.186	- .716	-1.8
$\$ \left(\frac{Q}{2} + R \right)$	66.554	65.284	-1.270	-1.9	162.824	159.944	-2.880	-1.8
ADD	32.5	29.9	-2.6	-8.0	46.0	45.3	- .7	-1.5
SSBUYS	7,996	7,571	- 425	-5.3	10,912	10,435	- 477	-4.4

TABLE I, based on a 20% sample of 1H and 1R cog items, shows significant improvements in annual costs and average days delay as a result of the proposed calculation. The change in investment $\left(\$ \left(\frac{Q}{2} + R \right) \right)$ represents a reduction in the value of buys during the transition between inventory policies.

Additional results indicate that for 1R cog, cost savings and ADD improvements result when the Negative Binomial (vice Normal) distribution is used for low demand items. Findings obtained by eliminating constraints on levels indicate that further cost-effectiveness can be obtained for both cogs by loosening unnecessary constraints.

Finally, the concept of pre-set (vice economically determined) P_{out} was tested to obtain an estimate of the resulting cost increases. Currently, SMA goals are set via a cog average, with price dependent variations for individual items. By pre-setting P_{out} , SMA for each item would be approximately the same, regardless of replacement price. The logic behind this concept is that holding P_{out} constant across a large group of items is equivalent to assuming that shortage cost is proportional to replacement price. The results indicate that at 85% SMA, annual costs would increase by 50%-75%.

5. Recommendations. The Navy Fleet Material Support Office (FMSO) recommends the following, regarding non-MARK O 1H and 1R cog items:

- . Implement the proposed consumable levels formulas which include shortage costs in the order quantity calculation and make direct use of the probability of being out of stock at a random point in time.
- . Use the Negative Binomial (vice Normal) distribution when computing the reorder level for 1R cog items whose leadtime demand is less than 20.
- . Remove or relax unnecessary constraints on the order quantity and reorder level calculation.

I. INTRODUCTION

Reference 1 established Department of Defense (DOD) policy for determining the order quantity (Q) and reorder level (R) for consumable items at the Inventory Control Points (ICPs). The mathematical model set forth by reference 1 attempts to minimize the total variable order and holding costs subject to a constraint on the time-weighted, essentiality-weighted requisitions short. In order to achieve this objective, optimal values for Q and R are sought for each item in the inventory. The Total Variable Cost (TVC) for the inventory is expressed as follows:

$$TVC = OC + HC + \lambda RS$$

where

OC = order costs

HC = holding costs

λRS = the implied cost of time-weighted, essentiality-weighted requisitions short

Reference 2 developed the formula for the reorder level currently used to conform to the policy established by reference 1. In reference 2, the total variable cost per year is expressed as follows:

$$\begin{aligned}
TVC = & \sum_{i=1}^N \frac{AD_i}{Q_i} + \sum_{i=1}^N IC_i \left(R_i + \frac{Q_i}{2} - \mu_i + \frac{1}{Q_i} \int_{R_i}^{\infty} (x - R_i) [F(x + Q_i; L) - F(x; L)] dx \right) \\
& + \lambda \sum_{i=1}^N \frac{E_i}{S_i Q_i} \int_{R_i}^{\infty} (x - R_i) [F(x + Q_i; L) - F(x; L)] dx
\end{aligned}$$

where

TVC = total variable cost

i = item index

N = item total

A = administrative order cost

D = mean annual demand

Q = order quantity

I = holding cost rate (obsolescence rate + storage rate + time preference rate)

C = replacement price

R = reorder level

μ = mean leadtime demand

x = variable of integration

F(·) = cumulative probability distribution of leadtime demand

L = procurement leadtime

λ = shortage cost per requisition short per year

E = item essentiality

S = requisition size

Minimization of total variable cost as expressed above requires

extensive computations. This is because of the interdependence of Q and R. Efforts to implement the mandated total variable cost function have employed approximations in order to segregate the decision variables. Reference 3 established that terms involving backorders would not affect the calculation of Q. Thus, the current order quantity formula is the Wilson Economic Order Quantity (Q_W), derived in reference 4:

$$Q_W = \sqrt{\frac{2AD}{IC}}$$

With Q established by reference 3, reference 2 compared methods of determining R. Reference 2 recommended a reorder level calculation which utilizes the optimal value of the probability of being out of stock at any point in time (P_{out}) as the optimal value of the probability of being out of stock during an order cycle (RISK). Reference 5 contains a detailed description of the current reorder level formula.

The (Q,R) calculation tested in this study follows directly from reference 6 and is intended to more closely follow reference 1. The proposed calculation results in levels without omitting the backorder terms in calculating Q and without using P_{out} to approximate the risk of stockout. Thus, the proposed calculation is theoretically more accurate and potentially more cost-effective. Within reference 6, the objective function from reference 2 is treated for individual items. Using the first derivative relative to R, the cost minimizing value of P_{out} , also written as P, is $\frac{SIC}{SIC + \lambda E}$. (This value is currently used as optimal risk.) Via the manipulations contained in APPENDIX B, the optimizing value of Q is calculated as $Q = \frac{Q_W}{\sqrt{1-P}}$. The optimal value of R is found

by solving an equation which relates P, Q, and R:

$$PQ = \int_R^{R+Q} [1 - F(x)] dx$$

This equation is derived from the equation $P = \int_R^{\infty} [F(x+Q) - F(x)] dx$,

which is developed in reference 2. The left side (PQ) of this equation may be interpreted as the optimal expected number of backorders in an order cycle. Thus, a unique value for R is found by setting the right side of this equation equal to the product of the previously derived values of P and Q. The mathematical development of the proposed (Q,R) calculations is contained in APPENDIX B.

An alternate method of using the optimal value of P_{out} as P_{out} and not as an approximation of the optimal risk was developed in reference 7. However, a preliminary investigation by COMNAVSUPSYSCOM indicated that the proposed approach was more promising. Hence, reference 6 tasked FMSO to evaluate the costs and benefits of the proposed change. Reference 8 forwarded the revised operations analysis study description.

This study compares the current and proposed levels calculations under various probability distribution and constraint settings. In the reorder level calculations, the assumed distribution of leadtime demand for non-MARK 0 items is determined by whether the mean leadtime demand exceeds the Negative Binomial breakpoint (Data Element Number (DEN) V028 - a user-determined parameter). If mean leadtime demand is lower than the parameter, the Negative Binomial distribution is chosen. Otherwise, the Normal distribution is used. Management practices vary from using the Normal distribution for all items to using the Negative Binomial distribution for all except

high demand (mean leadtime demand equals or exceeds 20) items. This study uses the latter practice as the base case. Similarly, the current and proposed calculations are compared under the practice of using the Normal distribution for all items. Also, because the proposed reorder level calculation is computationally infeasible when the order quantity is large and the assumed probability distribution is Negative Binomial, an additional parameter (QBREAKPOINT) is necessary in the proposed calculation. If the Negative Binomial distribution is selected on the basis of leadtime demand, the Normal distribution is then selected when the order quantity equals or exceeds QBREAKPOINT. This study includes a sensitivity analysis of different parameter values for QBREAKPOINT.

Since the proposed levels calculation includes backorder terms in the order quantity calculation and does not approximate optimal risk with optimal P_{out} , the proposed formulas are theoretically more accurate than the current levels calculation. The current Uniform Inventory Control Program (UICP) levels calculation includes constraints used in determining basic order quantity, constrained reorder level, constrained order quantity and acceptable procurement stockout risk. The order quantity is initially constrained to the basic order quantity by the discount quantity, shelf life, obsolescence, 12 quarters attrition demand, and one quarter attrition demand. The reorder level is constrained by maximum safety level, number of policy receivers, shelf life, obsolescence rate, maximum number of leadtimes' demand, and system reorder level low limit quantity. The basic order quantity is constrained by the discount quantity, shelf life, obsolescence rate, and safety level. Acceptable procurement stockout risk (P_{out} in the proposed calculation)

is constrained by managerially determined upper and lower bounds. (For details see APPENDIX C.) To establish whether the UICP constraints impede the effect of the proposed calculation, the study evaluates the effect of the proposed calculation under unconstrained and partially constrained conditions. The unconstrained case uses none of the constraints in APPENDIX C. The partially constrained case excludes all the levels constraints except those pertaining to risk (current calculation) and P_{out} (proposed calculation). In the partially constrained case, risk and P_{out} are constrained to be no more than 0.5. Additionally, the concept of variable shortage cost is explored. The current UICP levels approach assumes that the shortage cost does not vary across a large group of items. The constant shortage cost assumption causes Supply Material Availability (SMA) for individual items to vary approximately inversely with replacement price. An alternative concept is to set P_{out} constant across a large group of items (e.g., corresponding to a cognizance symbol), thus implying that shortage cost varies across the same group of items in proportion to replacement price. By pre-setting P_{out} , SMA for each item would be approximately the same, regardless of replacement price. An analysis was performed to test this concept and to estimate the increased total variable cost which results.

II. TECHNICAL APPROACH

The CARES III Analyzer, as documented by reference 9, was used to quantify the differences between the current and the proposed levels calculations. The CARES III program computes various statistical, financial, and performance data, thus permitting the evaluation of alternate inventory strategies. The CARES runs were made on 1H and 1R cog items for both the current and proposed calculations. The shortage cost parameters were adjusted to generate a system-wide steady state SMA of 85%, so that costs and Average Days Delay (ADD) could be compared at a constant performance level. The other system parameters were set consistent with current ICP management practices.

A. CARES MODIFICATIONS.

1. CARES was modified to perform either the current or proposed consumable levels calculation at the discretion of the user. The proposed calculation solves for (Q,R) via the following steps:

- a. Compute P_{out} as:

$$P = \frac{SIC}{SIC + \lambda E}$$

- b. Compute Q as:

$$Q = \frac{Q_W}{\sqrt{1-P}}$$

where

$$Q_W = \sqrt{\frac{2AD}{IC}}$$

c. Compute R as:

$$\text{Either } R = \text{Min} \left\{ X: Q - QP \leq \sum_{n=x}^{R+Q-1} \sum_{u=0}^n p(u) \right\}$$

(Negative Binomial distribution)

where

$p(u)$ = leadtime demand probability density function

$$\text{Or } R = \text{Min} \left\{ X: PQ \geq \int_X^{X+Q} [1 - F(X)] dx \right\}$$

(Normal distribution)

Details of the computational methods are contained in APPENDIX D.

2. CARES was modified to include or omit the UICP levels constraints described in APPENDIX C at the discretion of the user. CARES has the inherent capability to use unconstrained risk or P_{out} .

3. To facilitate computations, the Normal distribution is used to determine R when Q is large, even if the Negative Binomial distribution would be chosen on the basis of leadtime demand (leadtime demand less than Negative Binomial breakpoint). If Q is greater than or equal to the parameter QBREAKPOINT, the Normal (vice Negative Binomial) distribution is assumed to be the probability distribution for leadtime demand. QBREAKPOINT values of 15, 30, and 40 are tested in this study.

B. INPUT DATA. The input data for Navy Aviation Supply Office (ASO) and Navy Ships Parts Control Center (SPCC) were obtained from the Stratification (March 1980) and the Selected Item Generator (SIG) (September 1980) files, respectively. MARKs were recomputed and newly provisioned items and MARK 0 items were excluded. The 20% proportionally stratified (by MARK) samples of the remaining items resulted in 14,379 1R and 10,906 1H cog

items selected for analysis. The number of items used in each MARK is listed in TABLE I.

TABLE I
ITEM BREAKDOWN BY MARK

MARK	DESCRIPTION	NO. OF ITEMS	
		1H	1R
I	$D/4 \leq 5$, & $C \leq \$50$, & $(C)(D/4) \leq \$75$	2,760	3,652
II	$D/4 > 5$, & $C \leq \$50$, & $(C)(D/4) \leq \$75$	639	618
III	$D/4 \leq 5$, & $C > \$50$, or $(C)(D/4) > \$75$	5,352	6,700
IV	$D/4 > 5$, & $C > \$50$, or $(C)(D/4) > \$75$	2,155	3,409
TOTALS		10,906	14,379

C. PRINCIPAL COST-BENEFIT CONSIDERATIONS. With SMA held constant, the principal cost-benefit considerations are TVC and ADD. Since SMA is held constant for the study and the Navy holds shortage cost constant across a large group of items, total shortage cost does not vary between alternatives. Further, shortage cost values are currently implied by desired effectiveness levels and budget constraints rather than being an independently determined value. Hence, shortage cost is not a consideration for this study. Accordingly, the relevant Total Variable Cost is the sum of Procurement Order Cost (POC) and Holding Cost (HC).

1. Procurement Order Cost. For each cog, the procurement order cost was computed and summed over all the items in the sample. Decision rules identical to those in reference 5 are used to determine the procurement cost. Administrative order costs vary according to the value of the order quantity and whether negotiation of formal advertisement is used. The relevant DEN values are listed in TABLE II.

TABLE II
PROCUREMENT COSTS AND BREAKPOINT

DEN	DEFINITION	ASO	SPCC
V041	Low Value Annual Demand Order Cost	123	155
V042	Negotiated Procurement Order Cost	207	450
V043	Advertised Procurement Order Cost	207	500
V044	Low Value Annual Demand Breakpoint	7,500	2,000

The cost per procurement is multiplied by the average number of procurements per year. Thus, the annual procurement order cost is obtained for each item.

2. Holding Cost. Holding cost is based on the dollar value of the time-weighted average on-hand inventory. Average on-hand inventory equals average inventory position $\left(\frac{Q}{2} + R\right)$ minus average on-order quantity (μ), plus the expected backorder level at an arbitrary point in time (B). The time-weighted expected quantity on-order (expected leadtime demand at an arbitrary point in time) is omitted from the computation of holding costs since it does not vary with the inventory levels policy. In accordance with reference 1, the backorder term has little effect on the determination of inventory levels. Hence, this term is also omitted from cost computations. Therefore, the average inventory is computed as $\frac{Q}{2} + R$ for costing purposes. For each item, the quantity $\frac{Q}{2} + R$ was valued by C , the replacement price (B055), vice unit price (B053), to more sharply focus on investment costs dependent on

(Q,R) policy and to eliminate operational, e.g., transportation and pilferage, costs derived principally from customer and activity demand (B053-B055). The value of the average inventory is then multiplied by the holding cost rate which is the sum of storage cost (.01), time preference rate (.10), and obsolescence rate (.12). Thus, the holding cost for an item is computed as $.23(C)\left\{\frac{Q}{2} + R\right\}$.

3. Average Days Delay. Average days delay is the average time a requisition waits to be satisfied. The assumption was made that ICP processing time per requisition is not affected by the levels policy. Hence, ADD as calculated by CARES III for this study is based on delay for backordered requisitions only.

D. OTHER COST-BENEFIT CONSIDERATIONS. These quantities are generally considered to be subordinate to those listed under Paragraph C.

1. Transitional Effect on Value of Buys. The long-term annual value of buys is determined by system demand and, hence, is invariable relative to levels policies. However, during the transition between levels policies, the value of buys is affected by the initial inventory positions and by the changes in levels. The change in R implies a change in time until the first procurement, and the change in Q is the change in the quantity procured. The total transitional effect is equal to the approximate change in average inventory, or $\Delta(C)\left\{\frac{Q}{2} + R\right\}$. The transitional effect requires more than one year to take place. CARES III estimates the value of first year buys. But CARES does not directly state the value of buys for subsequent transition years. The change in the value of buys for subsequent years ($\Delta\$DYBUY$) is determined by subtracting the change in the value of

first year buys ($\Delta\$FYBUY$) from the total transitional change $\left(\Delta(C) \left(R + \frac{Q}{2} \right) \right)$.

2. Number of Procurements Per Year (SSBUYS). This is a measure of ICP workload.

3. Days Safety Level. Days safety level equals the value of safety level divided by the value of daily demand. The quantity is denoted as positive ($PSL(DAYS)$) or negative ($NSL(DAYS)$), depending on the sign of the safety level. Days safety level is of concern as a budgetary criterion.

III. FINDINGS

The results are presented in three main sections, described as follows:

- . SENSITIVITY OF THE PROPOSED CALCULATION TO QBREAKPOINT
- . LEVELS CALCULATION COMPARISONS
 - . Constrained Case
 - . Unconstrained Case
 - . Partially Constrained Case
 - . Comparisons between Constraint and Parameter Settings
- . VARIABLE SHORTAGE COST RESULTS

A. SENSITIVITY OF THE PROPOSED CALCULATION TO QBREAKPOINT. For large order quantities, the proposed reorder level calculation is infeasible when the Negative Binomial distribution is employed as the probability distribution of leadtime demand. A zero value for the Negative Binomial breakpoint forces the use of the Normal distribution for all items. The Negative Binomial breakpoint is 20 for the results in this section.

The QBREAKPOINT values tested in this study are 15, 30, and 40. TABLES III and IV show the results for 1H and 1R cogs in the constrained and unconstrained cases, respectively. In general, the results do not vary considerably between QBREAKPOINTS. Total Variable Cost and number of annual buys (SSBUYS) vary less than 0.5% between any two alternatives. ADD varies less than 1% between any two alternatives. Since the proposed reorder level calculation appeared insensitive to changes in QBREAKPOINT, a QBREAKPOINT value of 30 was arbitrarily chosen for this study.

TABLE III

EFFECT OF VARYING QBREAKPOINT UNDER
UICP CONSTRAINED LEVELS CALCULATION

	SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS					
	1H COG - 10,906 ITEMS			1R COG - 14,379 ITEMS		
	15	30	40	15	30	40
\$TVC	16.458	16.474	16.475	38.144	38.186	38.187
\$POC	1.458	1.458	1.458	1.398	1.399	1.399
\$HC	15.008	15.015	15.017	36.746	36.787	36.788
$\$ \left(\frac{Q}{2} + R \right)$	65.252	65.284	65.291	159.767	159.944	159.946
\$FYBUY	46.893	46.927	46.932	87.061	87.186	87.187
SSBUYS	7,567	7,571	7,571	10,427	10,435	10,436
ADD	30.0	29.9	29.8	45.6	45.3	45.3
PSL(DAYS)	89	89	90	78	79	79
NSL(DAYS)	71	71	71	0	0	0
ITEMS USING NORMAL	4,708	3,453	3,177	6,361	4,825	4,538

TABLE IV

EFFECT OF VARYING QBREAKPOINT
WITH UICP LEVELS CONSTRAINTS OMITTED

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS						
	1H COG - 10,906 ITEMS			1R COG - 14,379 ITEMS		
	15	30	40	15	30	40
\$TVC	15.309	15.326	15.327	31.130	31.035	31.031
\$POC	1.384	1.384	1.384	1.368	1.368	1.368
\$HC	13.925	13.942	13.943	29.762	29.667	29.663
$\$ \left(\frac{Q}{2} + R \right)$	60.542	60.616	60.622	129.400	128.987	128.970
\$FYBUY	43.235	43.285	43.289	68.017	67.814	67.802
SSBUYS	7,094	7,094	7,094	10,127	10,123	10,123
ADD	30.8	30.6	30.5	45.2	45.6	45.6
PSL (DAYS)	88	89	89	82	81	81
NSL (DAYS)	120	120	120	131	132	132
ITEMS USING NORMAL	5,216	4,129	3,768	7,278	5,548	5,143

B. LEVELS CALCULATION COMPARISONS.

1. Constrained Case. TABLES V and VI show the cost and performance results under the UICP constrained cases using both the Negative Binomial and Normal distributions and using only the Normal distribution for all items. In the first case, the Negative Binomial distribution is used when leadtime demand is less than 20. In each case, the proposed calculation causes a decrease in total variable cost, average days delay, and average inventory investment. TABLE V shows that when the Negative Binomial and Normal distributions are used, the decreases in total variable cost are \$.331 million and \$.716 million, respectively, for the samples of 1H and 1R cog items. Additionally, the procurement workload decreases in terms of the numbers of procurements for each cog because of the larger order quantities generated by the proposed calculation. The decrease is 5.3% and 4.4% for 1H and 1R cogs, respectively. A decrease in total procurement order costs also results, but not at the same rate as the decrease in number of buys. The difference is caused by higher administrative costs per procurement resulting from larger order quantities. TABLE V also shows that average inventory investment is reduced by \$1.27 million and \$2.88 million for the 1H and 1R cog samples, respectively. The reduction in inventory investment implies an equal reduction in the value of buys during the transition between inventory policies. The reduction in the value of buys equals $\Delta\$FYBUY$ (the dollar change in first year buys) plus $\Delta\$DYBUY$ (the delayed dollar change in buys). $\Delta\$DYBUY$ is obtainable only by subtraction of $\Delta\$FYBUY$ from $\Delta\$ \left(\frac{Q}{2} + R \right)$, since CARES does not state the value of buys for individual years beyond the first year. For example, for 1H

$\text{cog} - 1.270 = -.860 - .410$. In general, the transitional change in the value of buys is in the same direction as the change in total variable cost. This correlation results from the relatively high contribution (at least 90% for the runs in this study) of holding costs toward total variable cost per year. Since holding costs are $.23(C)\left(\frac{Q}{2} + R\right)$ and the change in $(C)\left(\frac{Q}{2} + R\right)$ equals the transitional change in the value of buys, the transitional cost effect is generally in the same direction as the annual effect. The decrease in ADD is 2.6 and 0.7 days for 1H and 1R cogs, respectively. Hence, for the base case (both the Negative Binomial and Normal distributions are used), implementing the proposed calculation would cause a considerable cost savings and a slight decrease in average days delay.

TABLE V

COMPARISON BETWEEN CURRENT AND PROPOSED LEVELS
CALCULATIONS UNDER THE NEGATIVE BINOMIAL/NORMAL DISTRIBUTIONS

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
1H COG - 10,906 ITEMS					1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	$\% \Delta$	CURRENT	PROPOSED		
\$TVC	16.805	16.474	- .331	- 2.0	38.902	38.186	- .716	- 1.8
\$POC	1.496	1.458	-3.800	- 2.5	1.452	1.399	- .053	- 3.7
\$HC	15.307	15.015	- .292	- 1.9	37.450	36.787	- .663	- 1.8
$\$ \left(\frac{Q}{2} + R \right)$	66.554	65.284	-1.270	- 1.9	162.824	159.944	-2.880	- 1.8
\$FYBUY	47.787	46.927	- .860	- 1.8	90.525	87.186	-3.339	- 3.7
\$DYBUY	N/A	N/A	- .410	N/A	N/A	N/A	.459	N/A
SSBUYS	7,996	7,571	- 425	- 5.3	10,912	10,435	- 477	- 4.4
ADD	32.5	29.9	-2.6	- 8.0	46.0	45.3	- .7	- 1.5
PSL(DAYS)	75	89	+ 14	+18.7	95	79	- 16	-16.8
NSL(DAYS)	36	71	+ 35	+97.2	0	0	0	N/A

N/A = Not Applicable

TABLE VI shows that when the Normal distribution is used for all items, the proposed levels calculation have a favorable impact on cost and inventory performance. The annual savings in \$TVC is \$.800 million and \$.697 million for the 1H and 1R cog samples, respectively. Procurement order costs and the number of buys also decrease due to the larger order quantities. As in TABLE V, the higher administrative order costs associated with larger order quantities prevent the total procurement order cost from decreasing at the same rate as the number of buys. The transitional decrease in buys is \$3.335 million and \$2.912 million for the 1H and 1R cog samples, respectively. The improvements in ADD are 1.1 and 1.4 days for 1H and 1R cogs, respectively. Hence, the use of the Normal distribution for all items does not substantially change the benefits of implementing the proposed calculation .

TABLE VI
COMPARISON BETWEEN CURRENT AND PROPOSED LEVELS
CALCULATIONS UNDER THE NORMAL DISTRIBUTION

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
1H COG - 10,906 ITEMS					1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	%Δ	CURRENT	PROPOSED	Δ	%Δ
\$TVC	18.332	17.532	- .800	-4.4	45.198	44.501	- .697	-1.5
\$POC	1.481	1.448	- .033	-2.2	1.675	1.647	-2.800	-1.7
\$HC	16.851	16.084	- .767	-4.6	43.523	42.853	- .670	-1.5
$S(\frac{Q}{2} + R)$	73.265	69.930	-3.335	-4.6	189.232	186.320	-2.912	-1.5
\$FYBUY	52.638	50.393	-2.245	-4.3	110.548	107.644	-2.904	-2.6
\$DYBUY	N/A	N/A	-1.090	N/A	N/A	N/A	- .008	N/A
SSBUYS	7,909	7,508	- 401	-5.1	12,721	12,462	-2.59	-2.0
ADD	26.5	25.4	-1.1	-4.2	108.3	106.9	-1.4	-1.3
PSL(DAYS)	94	100	+ 6	+6.4	219	205	- 14	-6.4
NSL(DAYS)	0	43	+ 43	N/A	0	0	0	N/A

N/A = Not Applicable

A comparison of TABLES V and VI indicates that the choice of Negative Binomial Breakpoint has a significant impact on costs. For 1H cog, using the Normal distribution for all items increases costs but reduces average days delay. For the 20% sample of 1H cog items, using the Negative Binomial distribution for low demand items saves \$1.0-1.5 million annually, depending on whether the current or proposed levels calculation is employed. However, using the Normal distribution for all 1H cog items reduces average days delay by 4.5-6.0 days, depending on the levels calculation employed. For 1R cog, using the Normal distribution for all items causes significant increases in both TVC (16-17%) and ADD (135-136%).

Of all the levels calculation and parameter settings tested in this section, the proposed levels calculation is the most advantageous regardless of the probability distribution policy employed. For 1H cog, the question of the most advantageous probability distribution policy depends on the relative value of cost and average days delay. For 1R cog, using the Negative Binomial distribution rather than the Normal distribution for low demand items is clearly the cost-effective choice. With costs shown in \$millions for the samples of each respective cog, the equally favorable calculation and probability distribution policies are:

a. 1H Cog.

(1) Proposed calculation with Negative Binomial/Normal distributions; TVC = 16.474, ADD = 29.9.

(2) Proposed calculation with Normal distribution only;
TVC = 17.532, ADD = 25.4.

b. 1R Cog.

(1) Proposed calculation with Negative Binomial/Normal distributions; TVC = 38.186, ADD = 45.3.

2. Unconstrained Case. The current and proposed levels calculations were tested under the same probability distribution policies employed in the constrained case. TABLES VII and VIII show the results pertaining to the respective policies. The data shows that total variable cost increases and average days delay decreases with the use of the proposed levels calculation, regardless of the probability distribution policy. TABLE VII shows that for the 1H and 1R cog samples, the proposed levels calculation would increase annual costs by \$.495 million and \$.488 million, respectively. The transitional buy effect reflects the increase in total variable cost. The corresponding decrease in average days delay is 2.8 and 1.8 days for 1H and 1R cog, respectively. The percentage decrease in steady-state buys is approximately 8% for each cog.

TABLE VIII shows similar results when the Normal distribution is used for all items, although the effect on 1R cog is more extreme than shown in TABLE VII. TABLE VIII shows that the annual cost increases by \$.715 million and \$8.462 million for the 1H and 1R cog samples, respectively. The decreases in average days delay are 2.1 and 14.6 days for 1H and 1R cogs, respectively. The considerable annual cost increases, reinforced by corresponding transitional buys, indicate that some degree of levels constraints are necessary for the proposed levels calculation to be clearly more cost-effective than the calculation currently in use.

TABLE VII

COMPARISON BETWEEN THE CURRENT AND PROPOSED UNCONSTRAINED LEVELS
CALCULATIONS UNDER THE NEGATIVE BINOMIAL/NORMAL DISTRIBUTIONS

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
	1H COG - 10,906 ITEMS				1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	$\% \Delta$	CURRENT	PROPOSED	Δ	$\% \Delta$
\$TVC	14.831	15.326	.495	3.3	30.547	31.035	.488	1.6
\$POC	1.395	1.384	-.011	0.0	1.462	1.368	-.094	-6.4
\$HC	13.436	13.942	.506	3.8	29.085	29.667	.582	2.0
$\$ \left(\frac{Q}{2} + R \right)$	58.416	60.616	2.200	3.8	126.455	128.986	2.532	2.0
\$FYBUY	41.606	43.285	1.679	4.0	65.335	67.814	2.479	3.8
\$DYBUY	N/A	N/A	.521	N/A	N/A	N/A	.053	N/A
SSBUYS	7,695	7,094	- 601	-7.8	11,026	10,123	- 903	-8.2
ADD	33.4	30.6	-2.8	-8.4	47.4	45.6	-1.8	-3.8
PSL(DAYS)	83	89	6	7.2	82	81	- 1	-1.2
NSL(DAYS)	107	120	13	12.1	126	132	6	4.8

N/A = Not Applicable

TABLE VIII

COMPARISON BETWEEN THE CURRENT AND PROPOSED UNCONSTRAINED LEVELS
CALCULATIONS UNDER THE NORMAL DISTRIBUTION

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
1H COG - 10,906 ITEMS					1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	% Δ	CURRENT	PROPOSED	Δ	% Δ
\$TVC	15.358	16.073	.715	4.7	30.385	38.847	8.462	27.8
\$POC	1.395	1.384	-.011	0.0	1.462	1.361	-.101	-6.9
\$HC	13.963	14.689	.726	5.2	28.923	37.486	8.563	29.6
$S\left(\frac{Q}{2} + R\right)$	60.708	63.866	3.158	5.2	125.751	162.982	37.231	29.6
\$FYBUY	43.338	45.418	2.080	5.8	82.366	92.816	10.450	12.7
\$DYBUY	N/A	N/A	1.079	N/A	N/A	N/A	26.781	N/A
SSBUYS	7,965	7,094	- 601	-7.8	11,026	10,065	- 961	- 8.7
ADD	28.8	26.7	-2.1	-7.3	71.0	56.4	-14.6	-20.6
PSL(DAYS)	98	99	1	1.0	197	230	33	16.8
NSL(DAYS)	102	102	0	0.0	241	122	- 119	-49.4

N/A = Not Applicable

Comparison between TABLE VII and TABLE VIII reveals the effect of probability distribution policy. For 1H cog the possibility exists to increase total variable cost and transitional buys for reduced average days delay. At the extremes are the current calculation with the Negative Binomial distribution used for low demand items (lowest cost, highest ADD) and the proposed calculation with the Normal distribution used for all items (highest cost, lowest ADD). The TVC difference is \$1.242 million for the sample of 1H cog items and a decrease by 6.7 in average days delay. The 1R cog items follow the same pattern indicated in the constrained case with regard to the probability distribution policy. The use of the Normal distribution for all items causes an increase in average days delay by at least 10 days without significantly decreasing costs. (The reduction from \$30.547 million to \$30.385 million under the current calculation is less than 1%, while the increase in ADD is 50%). Hence, for 1R cog, the best alternatives presented in this section employ the Negative Binomial distribution for low demand items. Under the current (vice proposed) levels calculation, the annual savings would be \$.488 million for the sample of 1R cog items with a decrease of 1.8 days delay. Showing the costs in \$millions for the samples of each respective cog, the equally favorable calculation and probability distribution policies are:

a. 1H Cog.

(1) Current calculation with Negative Binomial/Normal distributions; TVC = 14.831, ADD = 33.4.

(2) Proposed calculation with Negative Binomial/Normal distributions; TVC = 15.326, ADD = 30.6.

(3) Current calculation with Normal distribution only;

TVC = 15.358, ADD = 28.8.

(4) Proposed calculation with Normal distribution only;

TVC = 16.073, ADD = 26.7.

b. 1R Cog.

(1) Current calculation with Negative Binomial/Normal distributions; TVC = 30.547, ADD = 47.4.

(2) Proposed calculation with Negative Binomial/Normal distributions; TVC = 31.035, ADD = 45.6.

3. Partially Constrained Case. TABLE IX (Negative Binomial and Normal distributions) and TABLE X (Normal distribution only) show the results for the partially constrained case (risk and P_{out} are at most .5). As in the UICP constrained case, the most cost-effective parameter settings employ the proposed levels calculation. TABLE IX shows that when the Negative Binomial and Normal distributions are used, the decreases in total variable cost are \$.522 million and \$1.528 million for the samples of 1H and 1R cog items, respectively. Additionally, the procurement workload decreases in terms of the number of procurements for each cog, because of the larger order quantities generated by the proposed calculation. The decrease is 6.6% and 6.8% for 1H and 1R cogs, respectively. A decrease in total procurement order costs also results, but not at the same rate as the decrease in number of buys. The difference is caused by higher administrative costs per procurement resulting from higher order quantities. TABLE IX also shows that average inventory investment for the universe of items is reduced by \$2.289 million and \$6.338 million for the respective samples of 1H and 1R cogs. The reduction in inventory investment implies an

equal reduction in the value of buys during the transition between inventory policies. TABLE IX also shows a decrease in ADD by 1.8 and 0.6 days for 1H and 1R cogs, respectively. Hence, in the partially constrained case when the Negative Binomial and Normal distributions are used, implementing the proposed calculation would cause a considerable cost savings and a decrease in ADD.

TABLE IX

COMPARISON BETWEEN THE CURRENT AND PROPOSED PARTIALLY CONSTRAINED CALCULATIONS UNDER THE NEGATIVE BINOMIAL/NORMAL DISTRIBUTIONS

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
	1H COG - 10,906 ITEMS				1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	$\% \Delta$	CURRENT	PROPOSED	Δ	$\% \Delta$
\$TVC	16.610	16.088	- .522	-3.1	35.564	34.036	-1.528	-4.3
\$POC	1.395	1.399	.004	0.0	1.464	1.393	- .071	-4.8
\$HC	15.215	14.689	- .526	-3.5	34.100	32.643	-1.457	-4.3
$\$ \left(\frac{Q}{2} + R \right)$	66.154	63.865	-2.289	-3.5	148.262	141.924	-6.338	-4.3
\$FYBUY	47.073	45.781	-1.292	-2.7	79.994	77.567	-2.427	-3.0
\$DYBUY	N/A	N/A	- .997	N/A	N/A	N/A	-3.911	N/A
SSBUYS	7,695	7,178	- 510	-6.6	11,039	10,293	- 746	-6.8
ADD	31.9	30.1	-1.8	-5.6	44.5	43.9	-0.6	-1.3
PSL (DAYS)	78	87	9	11.5	74	76	2	0.2
NSL (DAYS)	38	81	43	113.2	29	67	38	131.0

N/A = Not Applicable

TABLE X shows that when the Normal distribution is used for all items, the proposed levels calculation has a favorable impact on both cost and ADD for 1H cog only. The annual savings is \$.928 million for the 1H cog sample. The procurement order cost and the number of buys decrease for both cogs as a result of the higher order quantities. As in the case in which the Negative Binomial distribution is used for low demand items (Negative Binomial breakpoint = 20), more costly procurements of higher order quantities prevent the total procurement order cost from decreasing at the same rate as the number of buys. The transitional decrease in buys for the 1H cog sample is \$4.062 million, and the decrease in ADD is 0.6 days. Hence, in the case of 1H cog, the use of the Normal distribution for low demand items does not substantially change the benefits of implementing the proposed calculation. For 1R cog, TABLE X shows that when the Normal distribution is used for all items, the ADD decreases by 9.0 days; however, the annual cost increases by \$.224 million. Hence, contrary to the case of 1H cog, when the Normal distribution is used for all 1R cog items, the proposed calculation requires an increase in annual costs for a decrease in ADD.

TABLE X

COMPARISON BETWEEN THE CURRENT AND PROPOSED PARTIALLY CONSTRAINED
CALCULATIONS UNDER THE NORMAL DISTRIBUTION

SMA = 85% - DOLLAR VALUES ARE IN \$MILLIONS								
	1H COG - 10,906 ITEMS				1R COG - 14,379 ITEMS			
	CURRENT	PROPOSED	Δ	$\% \Delta$	CURRENT	PROPOSED	Δ	$\% \Delta$
\$TVC	18.271	17.343	- .928	-5.1	42.260	42.484	.224	.01
\$POC	1.395	1.401	- .006	0.0	1.462	1.390	- .072	- 4.9
\$HC	16.876	15.942	- .934	-5.5	40.798	41.094	.296	.01
$\$ \left(\frac{Q}{2} + R \right)$	73.374	69.312	-4.062	-5.5	177.382	178.670	1.288	.01
\$FYBUY	52.406	49.784	-2.622	-5.0	99.387	103.616	4.229	4.3
\$DYBUY	N/A	N/A	-1.440	N/A	N/A	N/A	-2.941	N/A
SSBUYS	7,695	7,185	- 510	-6.6	11,026	10,261	- 765	- 6.9
ADD	25.8	25.2	-0.6	-2.3	63.4	54.4	-9.0	-14.2
PSL(DAYS)	99	100	1	0.0	181	222	41	22.7
NSL(DAYS)	0	48	48	N/A	0	41	41	N/A

N/A = Not Applicable

For 1R cog, a comparison of TABLES IX and X shows that the use of the Normal distribution for all items increases total variable costs for the 20% sample by \$5-10 million and increases ADD by 10.5 or 18.9 days, depending on the levels calculation. Hence, the most favorable case is shown in TABLE IX. Comparison between TABLES IX and X show that for 1H cog probability distribution policy may be used to trade reduced variable cost for reduced ADD. For each cog, the most cost-effective combinations of levels calculation and Negative Binomial breakpoint

settings employ the proposed calculation . Showing the costs in \$millions for the sample of each respective cog, the equally favorable calculation and probability distribution policies are:

a. 1H Cog.

(1) Proposed calculation with Negative Binomial/Normal distributions; TVC = 16.088, ADD = 30.1.

(2) Proposed calculation with Normal distribution only; TVC = 17.343, ADD = 25.2.

b. 1R Cog.

(1) Proposed calculation with Negative Binomial/Normal distributions; TVC = 34.036, ADD = 43.9.

4. Comparisons Between Constraint and Parameter Settings. Paragraph B.1 concludes that under the current UICP constraint settings the proposed levels calculation is more cost-effective than the current formulas. However, the parameter variations explored in this study reveal possibilities for further improvements in cost-effectiveness. TABLEs XI and XII display the critical data from the cases discussed in Paragraphs B.1,2,3 for 1H and 1R cogs, respectively. This section compares alternate constraint policies as well as the alternate probability distribution and levels calculation policies previously discussed.

a. 1H Cog. Inspection of TABLE XI permits comparison of alternate probability distribution policies and constraint settings, as well as levels calculations, based on the TVC and ADD criteria. The inclusion of average inventory investment permits the computation of relative transitional buy values.

Comparison of columns 2 and 4 with columns 1 and 3 supports the conclusion from Paragraph B.1 that under UICP constraints, the proposed levels calculation provides 85% SMA at lower total variable cost and with lower average days delay than the current levels calculation. Similarly, comparison of columns 10 and 12 with columns 9 and 11 shows the result from Paragraph B.3 that under the partially constrained conditions tested in this study the proposed levels calculation is more cost-effective than the current levels calculation. Further comparison shows that the fully constrained policy is less cost-effective than more unconstrained cases. Column 12 in relation to column 4 shows slightly lower TVC and ADD in the partially constrained case. Further, the difference in transitional buys is \$.618 million for the 1H cog sample. Comparison of column 7 with column 2 shows that when unconstrained, the current calculation with the Normal distribution used for all items provides 85% SMA with a decrease of 1.1 average days delay and a \$1.116 million savings in total variable cost for the 1H cog sample.

Most of the alternatives corresponding to columns 5, 6, 7, 8, 10, and 12 follow a pattern of increasing costs for decreasing ADD. Column 5 shows that the current unconstrained calculation using both the Negative Binomial and Normal distributions is the alternative with the least cost and highest ADD within the entire table. In contrast, column 12 shows that the proposed calculation, partially constrained and using the Normal distribution for all items, provides the least ADD of the tested alternatives and the highest cost of columns 5, 6, 7, 8, 10, and 12. The column 10 case is less cost-effective than the column 7 case, having both higher costs and ADD. The remaining alternatives include all four

TABLE XI

SUMMARY OF 1H COG RESULTS FOR 10,906 ITEMS
85% SMA COSTS ARE IN \$MILLIONS

LEVELS CALCULATION	1 CURRENT	2 PROPOSED	3 CURRENT	4 PROPOSED	5 CURRENT	6 PROPOSED	7 CURRENT	8 PROPOSED	9 CURRENT	10 PROPOSED	11 CURRENT	12 PROPOSED
NEGATIVE BINOMIAL BREAKPOINT	20	20	0	0	20	20	0	0	20	20	0	0
CONSTRAINTS	ALL UICP	ALL UICP	ALL UICP	ALL UICP	NONE	NONE	NONE	NONE	PARTIAL	PARTIAL	PARTIAL	PARTIAL
\$TVC	16.805	16.474	18.332	17.532	14.831	15.326	15.358	16.073	16.610	16.088	18.271	17.343
$\$ \left(\frac{Q}{2} + R \right)$	66.554	65.284	73.265	69.930	58.416	60.616	60.708	63.866	66.154	63.865	73.374	69.312
ADD	32.5	29.9	26.5	25.4	33.4	30.6	28.8	26.7	31.9	30.1	25.8	25.2
ANNUAL BUYS	7,996	7,571	7,909	7,508	7,695	7,094	7,965	7,094	7,695	7,178	7,695	7,185

NOTE: When leadtime demand is at least equal to the Negative Binomial breakpoint, the Normal distribution is used. Otherwise, the Negative Binomial distribution is used.

in the unconstrained case plus the partially constrained case in column 12.

Among these alternatives, increases in costs correspond to reduced ADD.

The results of the calculation, constraint, and parameter comparisons for 1H cog are summarized by the following observations:

- . For constrained and partially constrained cases, the proposed calculation is superior to the current calculation .
- . For constrained and partially constrained cases, use of the Negative Binomial distribution for low demand items is less costly but results in higher average days delay.
- . The current UICP constrained cases are not as cost-effective as less constrained cases.
- . The most cost-effective cases are :
 - .. the unconstrained cases regardless of probability distribution or levels calculation.
 - .. the partially constrained case with the proposed levels calculation and the Normal distribution only.

b. 1R Cog. Inspection of TABLE XII permits comparison of alternate probability distribution policies and constraint settings, as well as levels calculations, based on the criteria of TVC and ADD. The inclusion of average inventory investment permits the computation of relative transitional buy values.

Comparison of column 1 with column 3, column 2 with column 4, column 6 with column 8, column 9 with column 11, and column 10 with column 12 shows that using the Negative Binomial distribution for low demand items generally provides 85% SMA at lower cost and lower ADD than using the Normal distribution. The cost-effectiveness of the Negative Binomial distribution is further supported by comparing columns 5 and 7, where the increase in ADD (23.4 days) appears excessive considering the annual savings of \$0.162 million. Further inspection shows that for all the constrained or partially constrained cases, the proposed levels calculation is more cost-effective than the current levels calculation. Comparison of column 2 with column 10 shows that the partially constrained case provides a 1.4 day decrease in ADD with a transitional savings of \$18.020 million and an annual savings of \$4.150 million for the sample. The cases which are not dominated in cost-effectiveness by other alternatives are the unconstrained case using the proposed levels calculation and the Negative Binomial distribution for low demand items. Among these cases, increases in costs correspond to decreases in ADD.

The results of the calculation, constraint, and parameter variations for 1R cog are summarized with the following observations:

- . For constrained and partially constrained cases, the proposed calculation is superior to the current

TABLE XII

SUMMARY OF 1R COG RESULTS FOR 14,379 ITEMS
85% SMA COSTS ARE IN \$MILLIONS

LEVELS CALCULATION	1 CURRENT	2 PROPOSED	3 CURRENT	4 PROPOSED	5 CURRENT	6 PROPOSED	7 CURRENT	8 PROPOSED	9 CURRENT	10 PROPOSED	11 CURRENT	12 PROPOSED
NEGATIVE BINOMIAL BREAKPOINT	20	20	0	0	20	20	0	0	20	20	0	0
CONSTRAINTS	ALL UICP	ALL UICP	ALL UICP	ALL UICP	NONE	NONE	NONE	NONE	PARTIAL	PARTIAL	PARTIAL	PARTIAL
\$TVC	38.902	38.186	45.198	44.501	30.547	31.035	30.385	38.847	35.564	34.036	42.260	42.484
$\$ \left(\frac{Q}{2} + R \right)$	162.824	159.944	189.232	186.320	126.455	128.986	125.751	162.982	148.262	141.924	177.382	178.670
ADD	46.0	45.3	108.3	106.9	47.4	45.6	71.0	56.4	44.5	43.9	63.4	54.4
ANNUAL BUYS	10,912	10,435	12,721	12,462	11,026	10,123	11,026	10,065	11,039	10,293	11,026	10,261

NOTE: When leadtime demand is at least equal to the Negative Binomial breakpoint, the Normal distribution is used. Otherwise, the Negative Binomial distribution is used.

calculation, unless the Normal distribution is used for all items.

- . The use of the Normal distribution vice the Negative Binomial distribution for low demand items is more costly and results in higher ADD.
- . The current UICP constrained cases are not as cost-effective as less constrained cases.
- . The most cost-effective cases are:
 - .. the unconstrained case with the Negative Binomial distribution used for low demand items (current or proposed levels calculation).
 - .. the partially constrained case with the Negative Binomial distribution used for low demand items and the proposed levels calculation.

C. VARIABLE SHORTAGE COST RESULTS. In both the current UICP calculation and in the proposed calculation, the replacement price of an item is critical in determining the reorder level. In the current calculation, the reorder level is chosen based on the condition that risk equals $\frac{SIC}{SIC + \lambda E}$.

In the proposed calculation, the same quantity $\left(\frac{SIC}{SIC + \lambda E} \right)$ is set equal to P_{out} . In addition to the comparisons described under Paragraph B, CARES III runs were performed in which P_{out} was fixed, independently of replacement price, across the entire sample of items and the proposed levels calculation was employed.

Currently, SMA goals are set via a cog average, with price-dependent variations for individual items. By pre-setting P_{out} , SMA for each item would be approximately the same, regardless of price. When P_{out} is determined economically, lower priced items tend to obtain higher SMA than higher priced items. This is because higher P_{out} generally corresponds to lower SMA. (APPENDIX E shows the sensitivity of P_{out} to replacement price.) The economic determination of P_{out} follows directly from minimizing annual total variable cost relative to the reorder level. A distinctly different approach to determining P_{out} would, therefore, be expected to be less cost-effective in terms of SMA. Fixing P_{out} constant across a large group of items is such an approach. This approach has the advantage of reflecting shortage cost variations between items. In particular, the P_{out} formula transposes to λ equals $\frac{SIC (1-P)}{EP}$. Thus, holding P_{out} constant implies that the shortage cost is proportional to replacement price.

Total variable cost is indeed higher when P_{out} is pre-set. TABLES XIII - XVI show the results for 1H and 1R cogs. APPENDIX F contains graphs which show the same results, distinguished by Negative Binomial breakpoint, for the unconstrained case. The 85% SMA pre-set P_{out} cases

in TABLES XIII - XVI are derived from the graphs. The tables include economic P_{out} results at 85% SMA for comparison. The graphs pertaining to the 1H cog sample project total variable costs of \$25.6 and \$26.4 million for Negative Binomial breakpoints of 0 and 20, respectively. TABLES XIII and XIV show that for the unconstrained case with P_{out} determined economically, the TVC is less than \$16.5 million. Thus, the TVC increase caused by using pre-set P_{out} is over 50%. The graphs pertaining to 1R cog indicate that the TVC would be \$69 and \$46.5 million for Negative Binomial breakpoints of 0 and 20, respectively. TABLES XV and XVI show that for the unconstrained case, the TVC is approximately \$38.8 million and \$31.0 million for Negative Binomial breakpoints of 0 and 20, respectively when the economic P_{out} is employed. Thus, for 1R cog the cost increases by over 50%.

TABLE XIII

1H COG PRE-SET P_{out} RESULTS (NORMAL DISTRIBUTION ONLY)

10,906 ITEMS	P_{out}	SMA(%)	TVC (\$M)	ADD
Pre-Set P_{out} (Unconstrained)	.05	88	26.877	14.5
	N/A	85	25.600	N/A
	.15	76	22.006	36.5
	.25	66	19.106	66.0
Economic P_{out} (Unconstrained)	N/A	85	16.073	26.7
Pre-Set P_{out} (Constrained)	.15	75	22.025	36.9
Economic P_{out} (Constrained)	N/A	85	17.532	25.4

TABLE XIV

1H COG PRE-SET P_{out} RESULTS (NEGATIVE BINOMIAL/NORMAL DISTRIBUTIONS)

10,906 ITEMS	P_{out}	SMA(%)	TVC (\$M)	ADD
Pre-Set P_{out} (Unconstrained)	.05	88	28.39	17.4
	N/A	85	26.400	N/A
	.15	76	21.056	41.9
	.25	66	17.546	63.4
Economic P_{out} (Unconstrained)	N/A	85	15.326	30.6
Pre-Set P_{out} (Constrained)	.15	76	21.100	43.1
Economic P_{out} (Constrained)	N/A	85	16.474	29.9

N/A = Not Applicable

TABLE XV

1R COG PRE-SET P_{out} RESULTS (NORMAL DISTRIBUTION ONLY)

14,379 ITEMS	P_{out}	SMA (%)	TVC(\$M)	ADD
Pre-Set P_{out} (Unconstrained)	.05	93	84.361	15.2
	N/A	85	69.000	N/A
	.15	82	64.622	54.1
	.25	72	52.405	100.2
Economic P_{out} (Unconstrained)	N/A	85	38.847	56.4
Pre-Set P_{out} (Constrained)	.15	81	52.815	107.6
Economic P_{out} (Constrained)	N/A	85	44.501	106.9

TABLE XVI

1R COG PRE-SET P_{out} RESULTS (NEGATIVE BINOMIAL/NORMAL DISTRIBUTIONS)

14,379 ITEMS	P_{out}	SMA (%)	TVC(\$M)	ADD
Pre-Set P_{out} (Unconstrained)	.05	92	53.847	27.0
	N/A	85	46.500	N/A
	.15	81	44.410	45.3
	.25	65	38.666	65.2
Economic P_{out} (Unconstrained)	N/A	85	31.035	45.6
Pre-Set P_{out} (Constrained)	.15	83	45.829	44.6
Economic P_{out} (Constrained)	N/A	85	38.186	45.3

N/A = Not Applicable

It is clear from these comparisons that if a variable shortage cost concept is pursued in the form discussed herein, either cost would rise dramatically or SMA would decrease.

Employing pre-set P_{out} values provides the opportunity to compare SMA projected by the CARES III model with $1 - P_{out}$. For each cog, the difference between $1 - P_{out}$ and SMA is approximately constant across the tested range. TABLES XIII and XIV show the difference to be approximately nine percentage points of SMA for 1H cog. TABLES XV and XVI show the difference to be generally close to three percentage points of SMA for 1R cog. The exception to the latter rule is the case in which P_{out} equals .25 and the Negative Binomial breakpoint equals 20, where SMA drops to 10 percentage points below $1 - P_{out}$.

IV. SUMMARY AND CONCLUSIONS

This study quantifies the effects of employing the proposed levels calculation for consumable items. The proposed and current levels calculations are compared under several constraint and probability distribution policy combinations. The results show that under the current UICP constraints, the proposed levels calculation provides 85% SMA with reductions in both costs and ADD. The most cost-effective cases for all policy combinations are shown in TABLES XVII and XVIII for 1H and 1R cogs, respectively.

The data which is particularly relevant to the choice of levels formulas is discussed under Section III, Paragraph B.1 (UICP Constrained Case). To obtain the cost figures for the universes of 1H and 1R cog items, the cost figures for the 20% samples, shown under Paragraph III.B.1, are multiplied by 5. Thus, the annual cost savings for 1H cog would be \$1.655 million and \$4.0 million for the Negative Binomial/Normal and Normal only probability distribution policies, respectively. The transitional savings in these respective cases are \$6.35 million and \$16.675 million. Reinforcing the cost reductions, the respective days delay decreases are 2.6 and 1.1 days. For the 1R cog universe the annual cost savings would be \$3.580 and \$3.485 million for the Negative Binomial/Normal and Normal only probability distribution policies, respectively. The transitional savings in these respective cases are \$14.400 million and \$14.560 million, respectively. Reinforcing the cost reductions, the respective days delay decreases are 0.7 and 1.4 days.

An additional set of findings concerns the effect of probability

distribution policy (Negative Binomial breakpoint) on total variable cost and average days delay. Of particular interest is the reduction in both cost and average days delay for 1R cog items caused by using the Negative Binomial vice Normal distribution for low demand items (TABLES V and VI). This change would reduce annual costs by \$30-35 million, depending on which levels formulas are used. The corresponding reduction in average days delay would be 60-65 days. In the case of 1H cog, the probability distribution policy can be chosen in order to balance costs and ADD.

Further, analysis of data identified by constraint application (constrained, partially constrained, and unconstrained), as shown under Section III.B.4, yields the conclusion that the most effective cases are either partially constrained or unconstrained. Of particular interest are two partially constrained cases (P_{out} is no greater than 0.5), one for each cog. These cases are unique by being among the most cost-effective and by protecting the SMA of relatively high-priced items. Both cases use the proposed levels calculation. For 1H cog, the Normal distribution only is used; for 1R cog the Negative Binomial distribution is used for low demand items. The results contained herein showing the cost-effectiveness of unconstrained cases (all constraints on risk, P_{out} , order quantity, and reorder level are omitted) are consistent with the findings of references 10 and 11. References 10 and 11 discuss the value of specific constraints. Reference 10 concludes that specific constraints are of questionable theoretical or practical value.

Additionally, the concept of variable shortage cost was tested in several CARES III runs. Cost projections were made for various levels of SMA. Comparisons with the current constant shortage cost method shows the tested variable shortage cost method to be significantly more costly.

TABLE XVII

MOST COST-EFFECTIVE CALCULATION, CONSTRAINT, AND PARAMETER SETTINGS FOR 10,906
1H COG ITEMS (DOLLAR VALUES ARE IN \$MILLIONS)

LEVELS CALCULATION	CURRENT	PROPOSED	CURRENT	PROPOSED	PROPOSED
NEGATIVE BINOMIAL BREAKPOINT	20	20	0	0	0
CONSTRAINTS	NONE	NONE	NONE	NONE	PARTIAL
\$TVC	14.831	15.326	15.358	16.073	17.343
$\$ \left(\frac{Q}{2} + R \right)$	58.416	60.616	60.708	63.866	69.312
ADD	33.4	30.6	28.8	26.7	25.2

TABLE XVIII

MOST COST-EFFECTIVE CALCULATION, CONSTRAINT, AND PARAMETER SETTINGS FOR 14,379
1R COG ITEMS (DOLLAR VALUES ARE IN \$MILLIONS)

LEVELS CALCULATION	CURRENT	PROPOSED	PROPOSED
NEGATIVE BINOMIAL BREAKPOINT	20	20	20
CONSTRAINTS	NONE	NONE	PARTIAL
\$TVC	30.547	31.035	34.036
$\$ \left(\frac{Q}{2} + R \right)$	126.455	128.986	141.924
ADD	47.4	45.6	43.9

V. RECOMMENDATIONS

FMSO recommends the following regarding non-MARK 0 1H and 1R cog items:

- . Implement the proposed consumable levels formulas which include shortage costs in the order quantity calculation and make direct use of the probability of being out of stock at a random point in time.
- . Use the Negative Binomial (vice Normal) distribution when computing the reorder level for 1R cog items whose leadtime demand is less than 20.
- . Remove or relax unnecessary constraints on the order quantity and reorder level calculation.

APPENDIX A: REFERENCES

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4. Analysis of Inventory Systems, G. Hadley and T. M. Whitin
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Stocking Criteria and Levels Computation of 21 Jan 1981
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10. Operations Analysis Report 94
11. ALRAND Working Memorandum 387

APPENDIX B: DEVELOPMENT OF THE PROPOSED CALCULATION

This appendix develops a near-optimal solution for (Q,R) from a total variable cost per year equation. The contents are as follows:

SECTION 1 - Overview of Mathematical Development

SECTION 2 - Algebraic Relations which Support Section 1

SECTION 3 - Development of Computational Objectives for Determining
Reorder Points

SECTION 1 - OVERVIEW OF MATHEMATICAL DEVELOPMENT

TVC = Total Variable Cost per Year per Item

$$\frac{AD}{Q} + IC \left\{ \frac{Q}{2} + R - \mu + B \right\} + \frac{\lambda EB}{S}$$

where

Q = order quantity

R = reorder level

A = administrative order cost

D = annual demand in units

I = holding cost rate (obsolescence rate + storage rate + time preference rate)

C = replacement price

μ = mean leadtime demand

B(Q,R) = average backorder level at a random point in time

λ = shortage cost per requisition short per year

E = item essentiality

S = requisition size

Basic Approach.

Find (Q,R) such that the partial derivatives of TVC equal 0; i.e.,

$$(1) \frac{\partial(TVC)}{\partial R} = 0, \text{ and } (2) \frac{\partial(TVC)}{\partial Q} = 0. \text{ Thus, } \frac{\partial(TVC)}{\partial R} = IC + IC \frac{\partial B}{\partial R} + \frac{\lambda E}{S} \frac{\partial B}{\partial R} = 0.$$

Hence, condition (1) is equivalent to $\frac{IC}{IC + \lambda E/S} = \frac{-\partial B}{\partial R}$. Section 2 shows that

$\frac{-\partial B}{\partial R} = P$ = the probability of the system being out of stock at an arbitrary point in time. Hence, to meet condition (1), it is necessary to set

$$P = \frac{SIC}{SIC + \lambda E} \text{ . Section 3 of this appendix shows how this equation is used}$$

to obtain the reorder point.

Toward meeting condition (2), $\frac{\partial(\text{TVC})}{\partial Q} = \frac{-AD}{Q^2} + \frac{IC}{2} + IC \left(\frac{\partial B}{\partial Q} \right) + \frac{\lambda E}{S} \left(\frac{\partial B}{\partial Q} \right)$

Via the manipulations and approximations described in Section 2, one obtains $\frac{\partial(\text{TVC})}{\partial Q} = \frac{-AD}{Q^2} + \frac{IC}{2} - \frac{P^2}{2} \left(IC + \frac{\lambda E}{S} \right)$. Thus, to meet condition (2),

$$Q^2 = \frac{2AD}{IC(1-P^2) - P^2 \left(\lambda E/S \right)} \quad \text{With } P = \frac{SIC}{SIC + \lambda E} \text{ to meet condition (1)}$$

$$Q^2 = \frac{2AD}{IC \left\{ (1-P^2) - \frac{P^2(1-P)}{P} \right\}} = \frac{2AD/IC}{1-P} \quad \text{So, } Q = \sqrt{\frac{2AD/IC}{1-P}} = \frac{\text{WILSON EOQ}}{\sqrt{1-P}}$$

SECTION 2 - ALGEBRAIC RELATIONS WHICH SUPPORT SECTION 1

A. Development of $\frac{\partial B}{\partial R} = -P$.

B = expected backorder level at an arbitrary point in time

$b(t)$ = the expected backorder level a leadtime after the inventory position equals t

$f(x)$ = the probability density function of leadtime demand

$$\text{So, } b(t) = \int_t^{\infty} (x-t) f(x) dx.$$

If we assume that, between orders, inventory position varies linearly with time, we can write B as a position average. Thus,

$$B = \frac{1}{Q} \int_R^{R+Q} b(t) dt$$

$$\text{Hence, } \frac{\partial B}{\partial R} = \frac{1}{Q} \frac{\partial \left(\int_R^{R+Q} b(t) dt \right)}{\partial R} + \left(\int_R^{R+Q} b(t) dt \right) \frac{\partial \left(\frac{1}{Q} \right)}{\partial R} =$$

$$\frac{1}{Q} \frac{\partial \left(\int_R^{R+Q} b(t) dt \right)}{\partial R} + 0 =$$

$$\frac{1}{Q} \left\{ \int_R^{R+Q} \left(\frac{\partial b(t)}{\partial R} \right) dt + b(R+Q) \frac{\partial (R+Q)}{\partial R} - b(R) \frac{\partial R}{\partial R} \right\} =$$

$$\frac{1}{Q} \left\{ \int_R^{R+Q} \frac{\partial \int_t^{\infty} [(x-t) f(x) dx]}{\partial R} dt + b(R+Q) \frac{\partial (R+Q)}{\partial R} - b(R) \frac{\partial R}{\partial R} \right\} =$$

$$\frac{1}{Q} \{ b(R+Q) - b(R) \}$$

where

$b(R+Q)$ = the expected backorder level a leadtime from the time when the inventory position equals $R+Q$. So $b(R+Q)$ = the expected backorder level immediately following the arrival of an order.
 $b(R)$ = the expected backorder level a leadtime from when the inventory position equals R , and thus equals the expected backorder level immediately preceding the arrival of an order.

Hence, $b(R) - b(R+Q)$ = the expected number of backorders produced between orders. This latter quantity may also be expressed as total expected demand between orders multiplied by P_{out} . That is,

$$\left(\frac{\text{total expected demand per year}}{\text{number of orders per year}} \right) P = \frac{DQP}{D} = PQ$$

Hence, $b(R) - b(R+Q) = PQ$ and $\frac{\partial B}{\partial R} = \frac{-PQ}{Q} = -P$.

(A more rigorous discussion of P_{out} is available in reference 4.)

B. Development of $\frac{\partial(TVC)}{\partial Q} = \frac{-AD}{Q^2} + \frac{IC}{2} - \frac{P^2}{2} \left(IC + \frac{\lambda E}{S} \right)$.

From Section 1, $\frac{\partial(TVC)}{\partial Q} = \frac{-AD}{Q^2} + \frac{IC}{2} + IC \left(\frac{\partial B}{\partial Q} \right) + \frac{\lambda E}{S} \frac{\partial B}{\partial Q}$

$$\frac{\partial B}{\partial Q} = \frac{\partial \left\{ \frac{1}{Q} \int_R^{R+Q} b(t) dt \right\}}{\partial Q} = \frac{1}{Q} \frac{\partial \left\{ \int_R^{R+Q} b(t) dt \right\}}{\partial Q} + \int_R^{R+Q} b(t) dt \frac{\partial \left(\frac{1}{Q} \right)}{\partial Q} =$$

$$\frac{1}{Q} \left\{ \int_R^{R+Q} \frac{\partial b(t)}{\partial Q} dt + b(R+Q) \frac{\partial(R+Q)}{\partial Q} - b(R) \frac{\partial R}{\partial Q} \right\} - \frac{1}{Q^2} \int_R^{R+Q} b(t) dt =$$

$$\frac{1}{Q} \left\{ \int_R^{R+Q} \frac{\partial \left[\int_t^{\infty} (x-t)f(x)dx \right]}{\partial Q} + b(R+Q) \right\} - \frac{B}{Q} = \frac{b(R+Q) - B}{Q}$$

Under the simplifying assumptions that no backorders are carried between order cycles and that backorders occur during every order cycle,

$b(R+Q) = 0$, and hence, $\frac{\partial B}{\partial Q} = \frac{-B}{Q}$ and $B = \left(\frac{PQ}{2}\right) \left(\frac{PQ}{D}\right) \div \left(\frac{Q}{D}\right) = \frac{P^2 Q}{2}$. So,

$\frac{\partial B}{\partial Q} = -\frac{P^2}{2}$. Therefore, $\frac{\partial(TVC)}{\partial Q} = \frac{-AD}{Q^2} + \frac{IC}{2} - \frac{P^2}{2} \left(IC + \frac{\lambda E}{S} \right)$, which

is the form used in Section 1.

SECTION 3 - DEVELOPMENT OF COMPUTATIONAL OBJECTIVES FOR DETERMINING

REORDER POINTS

Section 1 determines the near optimal values for P , the probability of being out of stock at an arbitrary point in time, and Q , the order quantity. Via the equation

$$PQ = \int_R^{R+Q} [1 - F(x)]dx,$$

where $F(x)$ is the cumulative probability distribution of leadtime demand, a unique value of R is determined. Forms of this equation are discussed in references 2 and 4. The development of the computational objective varies slightly depending on whether the probability distribution of leadtime demand is discrete or continuous.

A. Negative Binomial Distribution. The discrete random variable version of the preceding equation is

$$PQ = \sum_{n=R}^{R+Q-1} \bar{F}(n),$$

where $\bar{F}(n) = P\{X > n\} = \sum_{u=n+1}^{\infty} p(u)$ and $p(u) = P\{X = u\}$.

Let $\sum_{n=R}^{R+Q-1} \bar{F}(n) = G(R)$. The following shows that $G(R)$ is a decreasing

function of R : $G(R+1) - G(R) = \sum_{n=R+1}^{R+Q} \bar{F}(n) - \sum_{n=R}^{R+Q-1} \bar{F}(n) = \bar{F}(R+1) + \bar{F}(R+2) + \dots +$

$$F(R+Q-1) + \bar{F}(R+Q) - [\bar{F}(R) + \bar{F}(R+1) + \dots + \bar{F}(R+Q-2) + \bar{F}(R+Q-1)] = \bar{F}(R+Q) - \bar{F}(R) < 0.$$

Hence, it is necessary to solve for

$$X = \text{Min} \{R: PQ \geq G(R)\} =$$

$$\text{Min} \left\{ R: PQ \geq \sum_{n=R}^{R+Q-1} \sum_{u=n+1}^{\infty} p(u) \right\} =$$

$$\text{Min} \left\{ R: PQ \geq \sum_{n=R}^{R+Q-1} \left[1 - \sum_{u=0}^n p(u) \right] \right\} =$$

$$\text{Min} \left\{ R: PQ \geq Q - \sum_{n=R}^{R+Q-1} \sum_{u=0}^n p(u) \right\} =$$

$$\text{Min} \left\{ R: Q(P-1) \geq - \sum_{n=R}^{R+Q-1} \sum_{u=0}^n p(u) \right\} =$$

$$\text{Min} \left\{ R: Q-QP \leq \sum_{n=R}^{R+Q-1} \sum_{u=0}^n p(u) \right\},$$

which is the computational objective used to modify CARES.

B. Normal Distribution. The applicable equation is

$$PQ = \int_R^{R+Q} [1 - F(x)] dx = H(R). \text{ The following shows that } H(R) \text{ is a decreasing}$$

$$\text{function of } R. \frac{\partial H(R)}{\partial R} = [1 - F(R+Q)] \frac{\partial(R+Q)}{\partial R} + \int_R^{R+Q} \frac{\partial[1-F(x)]}{\partial R} dx - [1 - F(R)] \frac{\partial R}{\partial R}$$

$$= 1 - F(R+Q) + 0 - 1 + F(R) = F(R) - F(R+Q) < 0. \text{ Hence, } \int_R^{R+Q} [1 - F(x)] dx$$

decreases as R increases, and the computational objective is to solve for

$$X = \text{Min} \left\{ R: PQ \geq \int_R^{R+Q} [1 - F(x)] dx \right\}.$$

APPENDIX C: LEVELS CONSTRAINTS

This appendix contains the UICP levels constraints and their application to the proposed levels calculation.

1. Acceptable Procurement Stockout Risk (ρ_1). The unconstrained procurement stockout risk (ρ) is constrained to ρ_1 as follows:

$$\rho_1 = \text{Min} \left\{ \begin{array}{l} \text{Max Risk} \\ \text{Max} \left\{ \begin{array}{l} \text{Min Risk} \\ \frac{\text{SIC}}{\text{SIC} + \lambda E} \end{array} \right\} \end{array} \right.$$

NOTE: ρ_1 is interpreted as risk in the current calculation and as P_{out} in the proposed calculation.

2. Basic Order Quantity (Q_1). The unconstrained order quantity (Q) is constrained to Q_1 as follows:

$$Q_1 = \left[\text{Max} \left[\begin{array}{l} \text{B061 } (D_2 - B_2) \quad \text{If B061} = 0, \text{ then use } 1 \\ \text{Min} \left\{ \begin{array}{l} 4(D_2 - B_2)L \\ \frac{4(D_2 - B_2)}{a} \\ 12(D_2 - B_2) \end{array} \right\} \\ \text{Max} \left\{ \begin{array}{l} (D_2 - B_2) \\ \left(\sqrt{\frac{8(A_1 + A_1')(D_2 - B_2)}{(i + s + a)C}} \right) \end{array} \right\} \end{array} \right] + .999 \right]^+$$

3. Constrained Reorder Level (XH). The unconstrained reorder level (X_1) is constrained to XH as follows:

$$XH = \left\{ \text{Max} \left[\begin{array}{l} 0 \\ \text{Min} \left\{ \begin{array}{l} \tilde{Z}_1 + xD_2 \\ \text{Max}(X_1, P_1) \\ 4L(D_2 - B_2) + \tilde{Z}_1 - I \\ 4(D_2 - B_2)/a + \tilde{Z}_1 - I \end{array} \right\} \\ K \tilde{Z}_1 \\ B020 \text{ (Set to 0 if } > 100,000) \end{array} \right] + .999 \right\}^+$$

4. Constrained Order Quantity (QH). QH is derived from Q_1 and XH, as follows:

$$QH = \text{Max} \left[\begin{array}{l} 1 \\ B061(D_2 - B_2) \quad \text{If } B061 = 0, \text{ use } 1 \\ \text{Min} \left\{ \begin{array}{l} Q_1 \\ 4L(D_2 - B_2) - \text{Max}(XH - \tilde{Z}_1; 0) \\ \frac{4(D_2 - B_2)}{a} - \text{Max}(XH - \tilde{Z}_1; 0) \end{array} \right\} + .999 \end{array} \right]^+$$

5. Source.

$$\rho = \frac{SIC}{SIC + \lambda E}$$

S = requisition size

I = holding cost rate ($i + s + a$)

i = time preference rate

s = storage cost

a = obsolescence rate

C = replacement price

λ = shortage cost per requisition short per year

E = item essentiality

$$Q = \sqrt{\frac{8(A_1 + A'_1)(D_2 - B_2)}{(i + s + a)C}}$$

A_1 = procurement order cost

A'_1 = manufacturer's set-up cost (B058)

D_2 = gross system demand-end of leadtime (B023D)

B_2 = system RFI regenerations - end of leadtime (B023F)

B061 = discount quantity

L = shelf life (C028)

\square^+ = largest integer function

X_1 = basic reorder level, computed so that risk = ρ_1 (current calculation)

or $P_{out} = \rho_1$ (proposed calculation)

\tilde{Z}_1 = procurement problem variable ($D_1 - B_1 + B_3$)

D_1 = gross system demand during leadtime (B023C)

B_1 = system RFI regenerations during leadtime (B023E)

B_3 = RFI regenerations during procurement problem average turn-around-time (B023G)

x = maximum number of quarters safety

I_1 = quantity per unit pack (C021B)

K = reorder level constraint rate (V295)

B020 = system reorder level low limit quantity

P_i = number of policy receivers

APPENDIX D: COMPUTATIONAL METHODS USED TO COMPUTE THE ORDER QUANTITY (Q)
AND REORDER POINT (X) UNDER THE PROPOSED CALCULATION

1. Order Quantity. The mathematical development leads to the equations

$$P = \frac{SIC}{SIC + \lambda E} \text{ and } Q = \sqrt{\frac{2AD/IC}{1-P}}$$

where

A = administrative order cost

D = mean annual demand (4(B074-B074A))

B074 = quarterly system demand forecast

B074A = quarterly system ready-for-issue regenerations forecast

I = holding cost rate (obsolescence rate + storage rate + time
preference rate)

C = replacement price (B055)

S = requisition size (B074/A023B)

A023B = system requisition average

λ = shortage cost per requisition short per year

E = item essentiality (C008C)

2. Reorder Level. As in the case of the current UICP model, the value of the procurement problem variable (\tilde{Z}) is the main indicator of which probability distribution is used in computing the reorder level (X). Since MARK 0 items were not included in the study, $\tilde{Z} < \text{Negative Binomial breakpoint}$ (DEN V028) implies that the Negative Binomial distribution is used. Otherwise,

the Normal distribution is used. However, a user determined parameter (QBREAKPOINT) was set so that when the order quantity is at least the specified value, the Normal distribution is used in order to conserve computer time.

a. Leadtime Demand Distribution Assumed Negative Binomial.

(1) When the probability distribution is Negative Binomial, the computational objective is to find

$$X = \text{Min} \left\{ X: Q - Qp_1 \leq \sum_{n=X}^{X+Q-1} \sum_{u=0}^n p(u) = \text{RHS}(X); X = 0, 1, 2, \dots \right\},$$

where p_1 (equivalent to P in Section 1) represents P (probability of being out of stock at random point in time) and $p(u)$ is computed as follows:

With

$$q = B019A/\tilde{Z}$$

$$\tilde{Z} = (B074)(B011A) - (B074A)(B011A) + (B074A)(B012F)$$

$$p = q - 1$$

$$K = \tilde{Z}/P$$

$$p(u) = \begin{cases} q^{-k}, & \text{for } u = 0 \\ \left(\frac{k+u-1}{u} \right) \left(\frac{p}{q} \right) p(u-1), & \text{for } u = 1, 2, 3, \dots \end{cases}$$

$\text{RHS}(X)$ may be interpreted as the expected quantity demanded and filled in the system per order cycle. This intuitively explains an essential feature of $\text{RHS}(X)$, which is that $\text{RHS}(X)$ increases as the reorder level increases.

(2) The following relations support the method used to compute reorder level.

Letting $RHS(X) = \sum_{n=X}^{X+Q-1} \sum_{u=0}^n p(u)$ and $C(J) = \sum_{u=0}^J p(u)$, it is clear that

$RHS(X) = C(X) + C(X+1) + \dots + C(X+Q-1)$, and $RHS(X+1) = C(X+1) + \dots + C(X+Q)$,

so, $RHS(X+1) - RHS(X) = C(X+Q) - C(X)$, and hence,

$RHS(X+1) = RHS(X) + C(X+Q) - C(X)$ and $C(N+1) = C(N) + P(N+1)$.

(3) The instructions for the computation of the reorder level under the proposed calculation, when the applicable leadtime demand distribution is Negative Binomial, are as follows. The notation is the same as that previously used in this appendix.

(a) Compute k and q .

(b) Initialize X , J , and u at 0.

(c) Let $p(u) = q^{-k}$, $C(J) = p(u)$, and $RHS(X) = C(J)$.

(d) If $Q=1$, go to instruction (h).

(e) Let $u = u+1$ and $J=J+1$.

(f) Let $p(u) = \left(\frac{k+u-1}{u} \right) \left(\frac{q-1}{q} \right) p(u-1)$, $C(J) = C(J-1) + p(u)$, and $RHS(X) = C(J) + RHS(X)$.

(g) If $J \neq Q-1$, go to instruction (e).

(h) If $RHS(X) \geq Q-Qp_1$, X is computed for this item under the specified parameter settings and the following steps are to be ignored.

(i) Let $X = X+1$, $J = J+1$, and $u = u+1$.

(j) If $X+Q-1 > 99$, use the Normal distribution. (This is to conserve computer core space and time.)

(k) Let $p(u) = \left(\frac{k+u-1}{u} \right) \left(\frac{q-1}{q} \right) p(u-1)$, $C(J) = C(J-1) + p(u)$ and $RHS(X) = RHS(X-1) + C(J) - C(X-1)$.

(l) If $RHS(X) < Q-Qp_1$, go to instruction (i).

(m) X is the reorder level for this item under the specified parameter setting.

b. Leadtime Demand Distribution Assumed Normal.

(1) When leadtime demand is assumed to follow a Normal distribution, the computational objective is to find

$$X = \text{Min} \left\{ X: Q\rho_1 \geq \int_X^{X+Q} \bar{F}(x)dx = \text{RHS}(X), X = 0, 1, 2, \dots \right\} \text{ where } \rho_1 \text{ is}$$

equivalent to P in Section 1 and $\bar{F}(x)$ is the reverse Normal distribution function with parameters $(\mu, \sigma^2) = (\tilde{Z}, B019A)$. $\text{RHS}(X)$ may be interpreted as the expected number of shortages just before the arrival of an order. This intuitively explains an essential feature of $\text{RHS}(X)$, which is that it decreases as the reorder level increases.

(2) The following manipulations show that

$$\text{RHS}(X) = \sigma \{ \phi(y_1) - \phi(y_2) + y_2 \Phi(y_2) - y_1 \Phi(y_1) \}$$

where

$\phi(y)$ = standard Normal probability density function

$\Phi(y)$ = reverse cumulative standard Normal probability distribution function

$$y_1 = \frac{X-\mu}{\sigma} \text{ and } y_2 = \frac{X+Q-\mu}{\sigma}, \text{ with } (\mu, \sigma) = (\tilde{Z}_1, \sqrt{B019A})$$

$$\text{RHS}(X) = \int_R^{R+Q} \bar{F}(x)dx = \int_R^{\infty} \bar{F}(x)dx - \int_{R+Q}^{\infty} \bar{F}(x)dx$$

$$\text{Let } y = \frac{x-\mu}{\sigma}; \text{ then } dy = \frac{dx}{\sigma}. \text{ Then, } \text{RHS}(X) = \sigma \int_{\frac{R-\mu}{\sigma}}^{\infty} \phi(y)dy - \sigma \int_{\frac{R+Q-\mu}{\sigma}}^{\infty} \phi(y)dy$$

To proceed further, it is necessary to prove the following equality:

$$\int_z^{\infty} \phi(y) dy = \phi(z) - z \phi(z)$$

Integrating by parts,

$$u = \phi(y) \quad du = -\phi(y) dy$$

$$dv = dy \quad v = y$$

Following the equation

$$uv = \int u dv + \int v du, \text{ and transposing,}$$

$$y \phi(y) + \int y \phi(y) dy = \int \phi(y) dy$$

Consider the middle term.

$$\int y \phi(y) dy = \int y \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy$$

$$\text{Let } u = \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} = \phi(y)$$

$$du = \frac{-ye^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy = -y \phi(y) dy$$

$$\int du = u \rightarrow \int y \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy = -\phi(y)$$

$$\text{Then, } \int_z^{\infty} \phi(y) dy = (y \phi(y) - \phi(y)) \Big|_z^{\infty}$$

$$= y \phi(y) \Big|_z^{\infty} - \phi(y) \Big|_z^{\infty} = 0 - z \phi(z) - 0 + \phi(z) = \phi(z) - z \phi(z)$$

Thus, the equality is proved.

Substituting this result into RHS(X), with $y_1 = \frac{X-\mu}{\sigma}$ and $y_2 = \frac{X+Q-\mu}{\sigma}$,

$$\begin{aligned} \text{RHS}(X) &= \sigma \left\{ \int_{y_1}^{\infty} \phi(y) dy - \int_{y_2}^{\infty} \phi(y) dy \right\} = \\ &= \sigma \{ \phi(y_1) - y_1 \Phi(y_1) - \phi(y_2) + y_2 \Phi(y_2) \} = \\ &= \sigma \{ \phi(y_1) - \phi(y_2) + y_2 \Phi(y_2) - y_1 \Phi(y_1) \}. \end{aligned}$$

Thus, RHS may be computed by using the approximation

$$\begin{aligned} \Phi(y) &= \phi(y) [b_1 Z - b_2 Z^2 + b_3 Z^3] \text{ with } Z = \frac{1}{1+pt}, \quad b_1 = .4361836 \\ & \quad b_2 = .1201676 \\ & \quad b_3 = .9372980 \\ & \quad p = .33267 \end{aligned}$$

(3) Guiding principles for the computational method used in the project computer program are as follows:

(a) To conserve CPU space and time, use the following shortcuts. When either y_1 or y_2 are < -2.33 , set $\phi(y)$ and $\Phi(y)$ to .026 and .99, respectively. When either y_1 or y_2 are > 2.33 , set $\phi(y)$ and $\Phi(y)$ to .026 and .01, respectively.

(b) Boundaries for possible values of X will be established by comparison.

(c) If the upper bound (EST2) minus the lower bound (EST1) is less than or equal to 3, the interval will not be further divided.

(d) If the preceding condition applies, $\text{RHS}(X)$ for consecutively increasing values of X , starting with $X = \text{EST1}+1$ (or if $Qp_1 \geq \text{RHS}\left(\left\lceil \frac{Q}{2} \right\rceil\right)$, $X = 0$), will be computed and compared to Qp_1 .

(e) If the upper bound (EST2) minus the lowest bound (EST1) is greater than 3, the interval will be divided roughly in half, with the middle value denoted by EST3 .

(f) $\text{RHS}(\text{EST3})$ will be compared to Qp_1 and EST3 will become either EST1 or EST2 , as appropriate.

(g) Notation: $[y] =$ largest integer less than or equal to y (e.g., $[9.8] = [9.3] = 9$).

(4) The instructions for the computation of X are as follows:

(a) Compute $\text{RHS}(\lceil \tilde{Z} \rceil)$.

(b) If $\text{RHS}(\lceil \tilde{Z} \rceil) \leq Qp_1$, let $\text{EST1} = -1$ and $\text{EST2} = \lceil \tilde{Z} \rceil$, and go to instruction (g).

(c) Let $M = 3$.

(d) Compute $\text{RHS}\left(\left\lceil \frac{M\tilde{Z}}{2} \right\rceil\right)$.

(e) If $\text{RHS}\left(\left\lceil \frac{M\tilde{Z}}{2} \right\rceil\right) \leq Qp_1$, let $\text{EST1} = \left\lfloor \frac{(M-1)\tilde{Z}}{2} \right\rfloor$ and $\left\lceil \frac{M\tilde{Z}}{2} \right\rceil = \text{EST2}$ and go to instruction (g).

(f) Let $M = M + 1$ and go to instruction (d).

(g) If $\text{EST2} - \text{EST1} \leq 3$, let $J = \text{EST1} + 1$ and go to instruction (l).

(h) Let $\text{EST3} = \left\lfloor \frac{\text{EST1} + \text{EST2}}{2} \right\rfloor$.

(i) Compute $\text{RHS}(\text{EST3})$.

(j) If $Qp_1 \geq \text{RHS}(\text{EST3})$, let $\text{EST2} = \text{EST3}$ and go to instruction (g).

(k) Let $\text{EST1} = \text{EST3}$ and go to instruction (g).

(l) Compute $RHS(J)$.

(m) If $RHS(J) > Q\rho_1$, let $J = J + 1$ and go to instruction (l).

(n) Let $X = J$. X is the reorder level for this item under the specified parameter setting.

APPENDIX E: SENSITIVITY OF P_{out} AND RISK TO REPLACEMENT PRICE

1. Algebraic Effect of B055 on Economically Determined P_{out} (RISK).

General formula for P_{out} (proposed calculation) and RISK (current calculation):

$$P_{out} = P = \frac{SIC}{SIC + \lambda E}$$

where

S = requisition size (B074/A023B)

B074 = quarterly system demand forecast

A023B = system requisition average

I = holding cost rate (obsolescence rate + storage rate + time preference rate)

C = replacement price (B055)

λ = shortage cost per requisition short per year

E = item essentiality (C008C)

Derivatives:

$$\frac{\partial P}{\partial C} = \frac{(SIC + \lambda E) SI - (SIC) SI}{(SIC + \lambda E)^2} =$$

$$\left(\frac{SI}{(SIC + \lambda E)} \right) \left(\frac{\lambda E}{SIC + \lambda E} \right) =$$

$$\left(\frac{P}{C} \right) \left(\frac{P \lambda E}{SIC} \right) = \frac{P^2 \lambda E}{SIC}$$

$$\frac{\partial^2 P}{\partial C^2} = \frac{(SIC + \lambda E)^2 (0) - (SI \lambda E)^2 (SIC + \lambda E) SI}{(SIC + \lambda E)^4} =$$

$$\frac{-2(SIC + \lambda E) (SI)^2 \lambda E}{(SIC + \lambda E)^4} = \frac{-2(SI)^2 \lambda E}{(SIC + \lambda E)^3} =$$

$$\frac{-2(SI)^2 \lambda E}{(SIC + \lambda E)^2 (SIC + \lambda E)} = \frac{-2 P^2 \lambda E P}{C^2 (SIC)} =$$

$$\frac{-2 P^3 \lambda E}{C^3 SI}$$

Thus, the behavior of P as a function of B055 may be described as logarithmic, insofar as P increases as a function of B055, at a decreasing rate, which decreases slower as B055 becomes larger.

The derivatives of $y = \ln x$ indicate the aforementioned features:

$$\frac{\partial y}{\partial x} = \frac{1}{x} ; \quad \frac{\partial^2 y}{\partial x^2} = -\frac{1}{x^2}$$

2. Numerical Sensitivity.

Formula: $P = \frac{SIC}{SIC + \lambda E}$

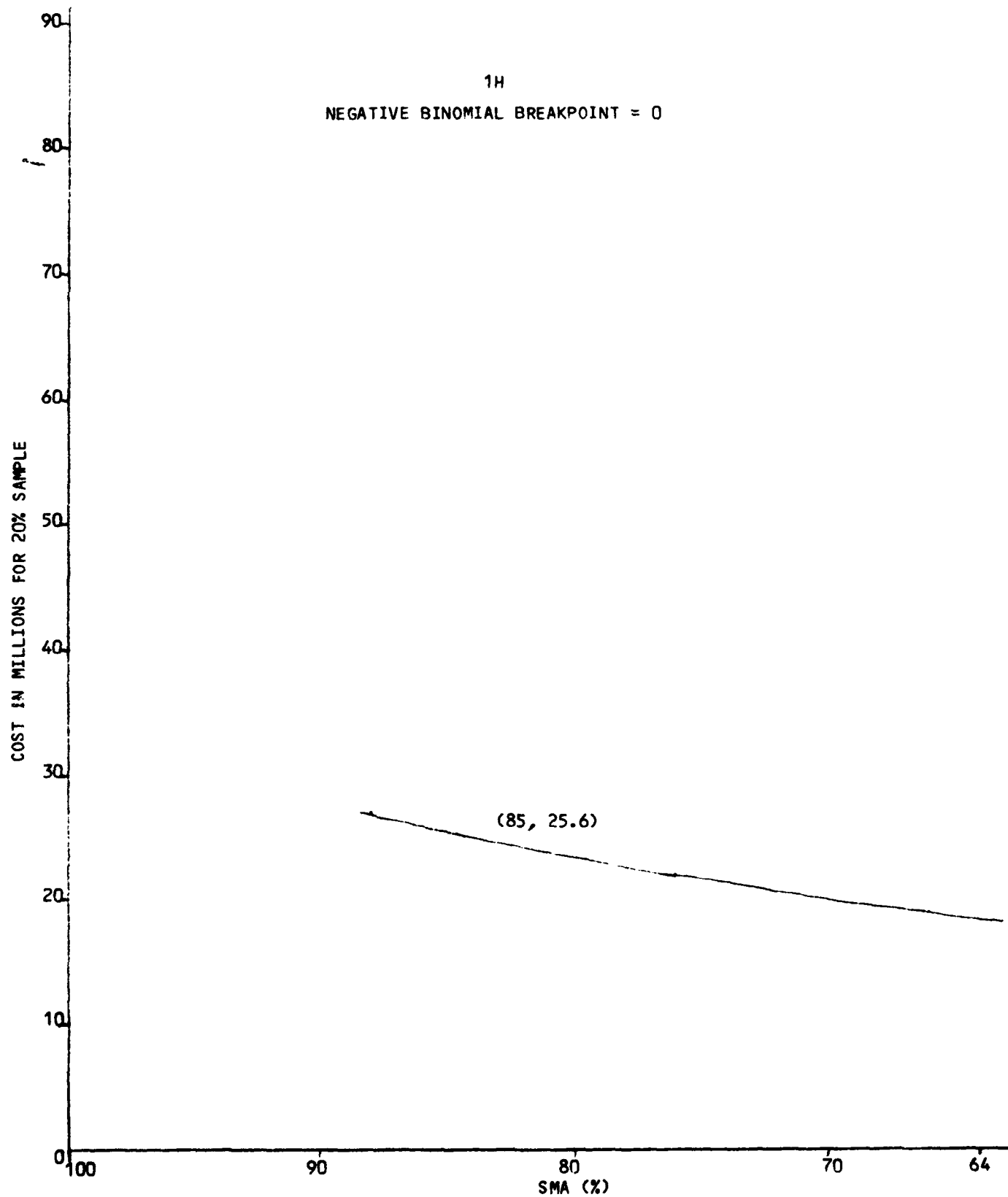
P_{out} VALUES

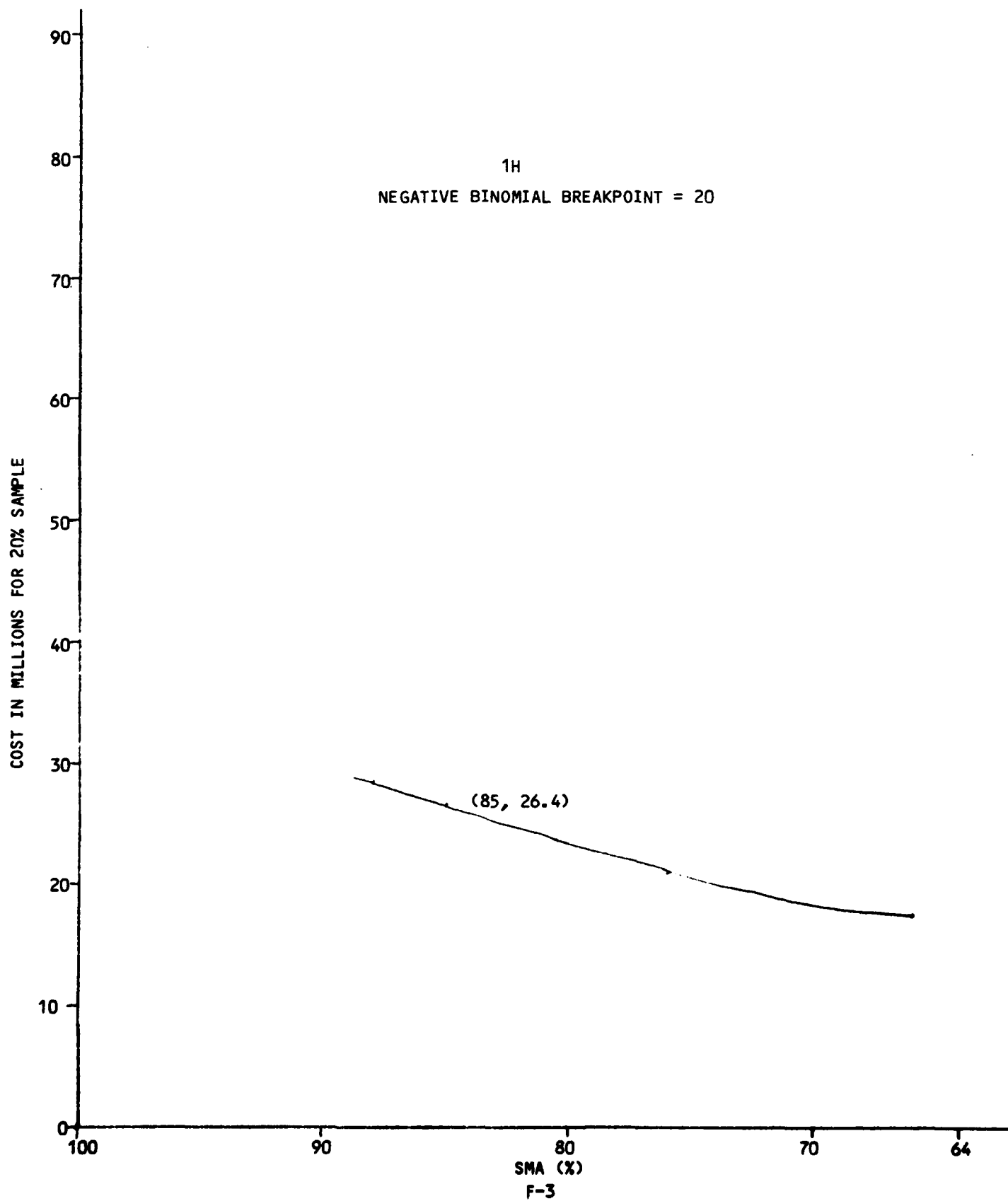
B055(\$)	$\lambda E = 500$		$\lambda E = 1000$	
	S=1	S=5	S=1	S=5
1	.00	.00	.00	.00
10	.00	.02	.00	.01
50	.02	.10	.01	.05
100	.04	.19	.02	.10
500	.19	.53	.10	.37
1000	.32	.70	.19	.53
2000	.48	.82	.32	.70
3000	.58	.87	.41	.78
4000	.65	.90	.48	.82
5000	.70	.92	.53	.85

APPENDIX F: PRE-SET P_{out} GRAPHS

The enclosed graphs (described below) project total variable cost for 10,906 1H cog items and 14,379 1R cog items at 85% SMA, when P_{out} is pre-set via economically determined. UICP constraints were omitted in the calculations.

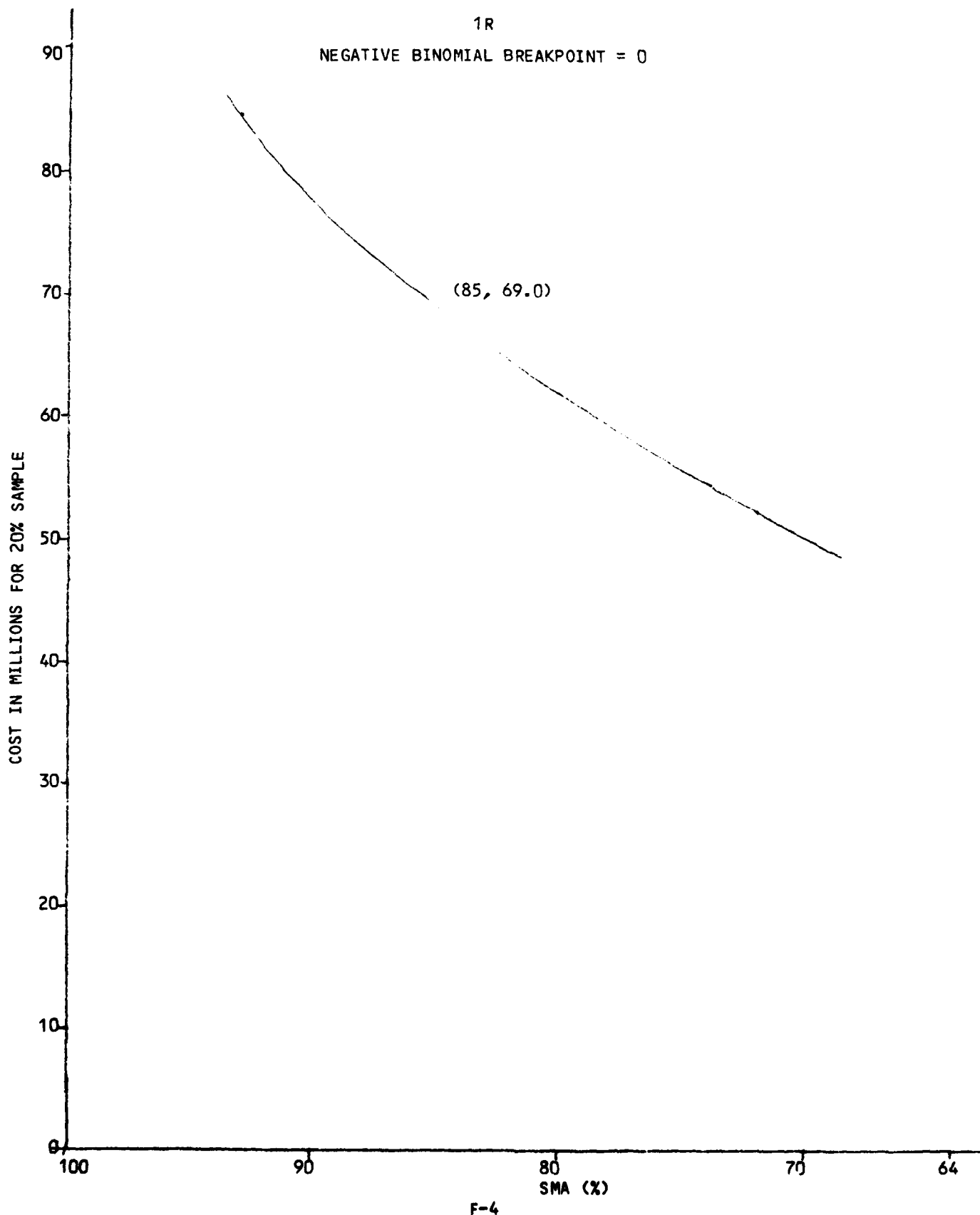
- . GRAPH 1 - 1H Cog with Normal distribution only
- . GRAPH 2 - 1H Cog with Negative Binomial/Normal distributions
- . GRAPH 3 - 1R cog with Normal distribution only
- . GRAPH 4 - 1R cog with Negative Binomial/Normal distributions





1R

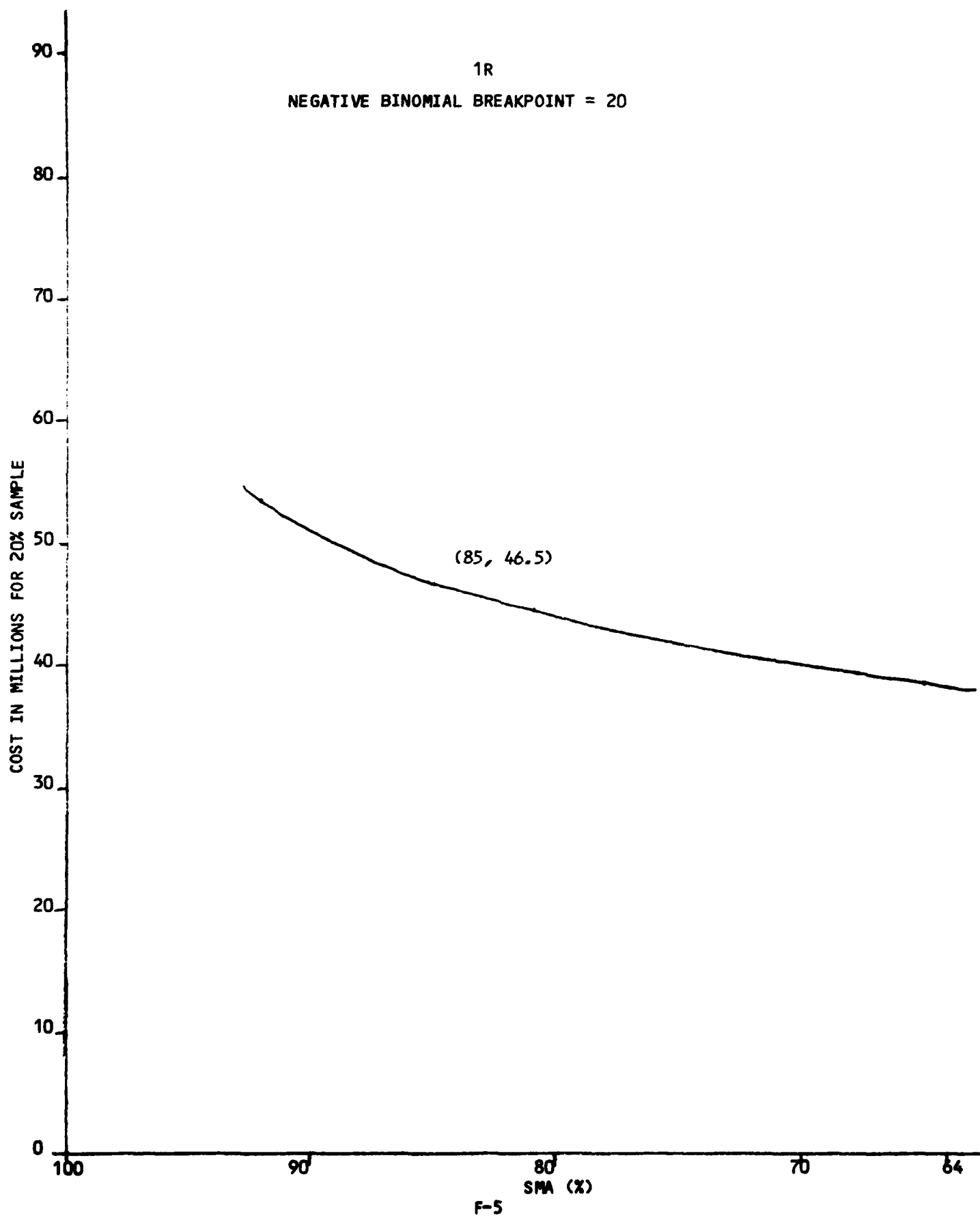
NEGATIVE BINOMIAL BREAKPOINT = 0



F-4

1R

NEGATIVE BINOMIAL BREAKPOINT = 20



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13. ABSTRACT <p>This study is a cost-benefit analysis of a proposed levels (order quantity and reorder level) calculation for consumable items. The Computation and Research Evaluation System (CARES) III Analyzer was used to evaluate the proposal. The input data consisted of samples of 1H and 1R cognizance (COG) items. The criteria for evaluation were Total Variable Cost and Average Days Delay. Since the Navy imputes shortage cost from desired performance and budget levels, the relevant Total Variable Cost is the sum of procurement order cost and holding cost. Supply Material Availability was held constant at 85%. Additional cost projections were obtained concerning the implementation of a variable shortage cost.</p> <p>This study makes the following recommendations regarding non-MARK O 1H and 1R cog items:</p> <ul style="list-style-type: none">• Implement the proposed consumable levels formulas which include shortage costs in the order quantity calculation and make direct use of the probability of being out of stock at a random point in time.• Use the Negative Binomial (vice Normal) distribution when computing the reorder level for 1R cog items whose leadtime demand is less than 20.• Remove or relax unnecessary constraints on the order quantity and reorder level calculation.			

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Associate Professor, Code 54 Mg
Naval Postgraduate School
Monterey, CA 93940

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